GETT Correspondence Paper 4.2: General Relativity Correspondence II: Einstein-Domain Dynamics and Field-Equation Recovery

Author: John Edward Holland **ORCID:** 0009-0001-5120-8712

Email: john.holland@expansetension.org **Date:** 2nd April 2026

A Domain-Limited Derivation of Einstein Field Equations from Coarse-Grained Φ–Matter Dynamics

Abstract

This paper establishes the dynamical correspondence between General Expanse Tension Theory (GETT) and General Relativity (GR) within a precisely defined physical regime. Building on the geometrisation limit derived in Paper 4.1, we demonstrate that the coarse-grained dynamics of a real, covariant singlet scalar field Φ, coupled to matter via a density-dependent interaction, admit a closed macroscopic description in which spacetime curvature evolves according to Einstein-domain equations.

Starting from first-principles Φ-field dynamics, we construct the effective stress-energy tensor of the coupled matter–substrate system and show that it is locally conserved under controlled coarse-graining. We then define explicit closure conditions – including mid-density coupling stability, weak coupling variation, smooth field behaviour, adiabatic evolution, and effective universality – under which the emergent metric satisfies a curvature–source relation of the form

$$G_{\mu\nu} = 8\pi G_{\text{eff}}\, T_{\mu\nu}^{\text{eff}} + \Delta_{\mu\nu}$$

where all non-Einstein contributions are isolated in a physically interpretable correction tensor $\Delta_{\mu\nu}$.

We show that, within the Einstein domain, these corrections are perturbatively suppressed, yielding recovery of the Einstein field equations to leading order, with exact correspondence in the formal limit of vanishing control parameter. The effective gravitational coupling emerges from the underlying Φ–matter interaction and is shown to be approximately constant within this regime.

This result establishes General Relativity as a domain-limited dynamical closure of an underlying scalar-field substrate, providing a physically grounded reconstruction in which both the success and the limitations of GR arise from explicit, testable conditions. The work reframes gravity from a fundamentally geometric theory to an emergent macroscopic manifestation of substrate dynamics, delivering a unified framework that not only reproduces Einstein's equations but explains why and where they apply.

Keywords: gravitation, general relativity, scalar field theory, emergent gravity, effective field theory, coarse-graining, modified gravity, galaxy dynamics, cosmology, theoretical physics.

1. Introduction

1.1 Purpose and Context

General Relativity (GR) provides the most successful macroscopic description of gravitation, representing gravitational phenomena through the dynamics of spacetime geometry. In this framework, the Einstein field equations relate curvature to matter–energy content, forming a closed dynamical system that has been extensively validated across a wide range of physical regimes [1-3,5-7]. A system of equations is *closed* when all quantities appearing in the equations can be expressed in terms of variables within the system itself, without requiring additional external information. However, GR is formulated as a geometric theory, in which curvature is taken as fundamental. It does not specify an underlying physical mechanism from which this geometric structure arises, nor does it define the conditions under which its dynamical form should be expected to hold or fail.

The General Expanse Tension Theory (GETT) correspondence programme addresses this limitation by reconstructing established physical theories as **domain-limited effective descriptions** of an underlying physical substrate. In this framework, gravitational phenomena emerge from the dynamics of a real, covariant scalar field Φ, coupled to matter through a density-dependent interaction. Paper 4.1 [29] established that, under appropriate physical conditions, the coarse-grained Φ–matter system admits an effective metric description, thereby recovering the **geometrisation limit** of GR. The present paper extends this result from kinematics to dynamics.

Previous emergent-gravity approaches typically postulate geometric structure or thermodynamic identities, whereas the present work derives Einstein-domain dynamics from an explicit microscopic action with controlled coarse-graining and defined domain of validity.

1.2 Core Objective

The objective of this paper is to demonstrate that, within a precisely defined physical regime, the effective metric arising in GETT obeys Einstein-domain dynamical equations. Specifically, we aim to show that:

- the coarse-grained Φ–matter system admits a well-defined, conserved effective stress-energy tensor,
- the emergent metric supports a consistent curvature structure,
- and the resulting dynamics admit a closure of the form

$$G_{\mu\nu} = 8\pi G_{\text{eff}}\, T_{\mu\nu}^{\text{eff}} + \Delta_{\mu\nu}$$

with the correction tensor $\Delta_{\mu\nu}$ explicitly identified and perturbatively suppressed within the Einstein domain. The central claim is therefore:

Einstein's field equations arise as the leading-order dynamical closure of the coarse-grained Φ–matter system under smooth, weakly varying, adiabatic, and effectively universal conditions.

1.3 Approach and Methodology

The approach adopted in this work is deliberately constructive and conditional, following the discipline established in the GETT Correspondence Series Paper 3 Special Relativity [28]. Rather than assuming the validity of Einstein's equations, we:

1. **Define formal correspondence requirements** (Section 2), specifying the structural, functional, conservation, precision, and domain criteria that must be satisfied for valid GR correspondence.
2. **Establish the first-principles substrate dynamics** (Section 3), including:
   - the governing equation of the Φ field,
   - its energy–momentum content,
   - and its density-dependent coupling to matter.
3. **Introduce a controlled coarse-graining procedure** (Section 4), mapping microscopic substrate dynamics to macroscopic continuum variables compatible with GR.
4. **Construct a unified effective stress-energy tensor** (Section 5), incorporating matter, Φ-field, and interaction contributions, and demonstrate its conservation at the effective level.
5. **Define the conditions for Einstein-domain closure** (Section 6), expressed as explicit inequalities governing:
   - coupling variation,
   - Φ-field smoothness,
   - temporal evolution,
   - and universality of matter response.
6. **Derive the Einstein-form dynamical equations** (Section 7), showing that:
   - curvature tensors constructed from the emergent metric couple to the effective source,
   - all non-Einstein contributions are isolated in a correction tensor,
   - and these corrections vanish in the Einstein-domain limit.

At each stage, no structure is assumed without derivation, and all approximations are explicitly stated and controlled.

1.4 Methodological Standard

This paper adheres to a strict correspondence standard:

- **No geometric structure is postulated independently of the Φ-substrate.**
- **No conservation law is assumed without derivation from underlying dynamics.**
- **No approximation is introduced without explicit domain conditions.**
- **All deviations from GR are identified, classified, and parameter-controlled.**

Correspondence is therefore not asserted but **demonstrated against explicit acceptance criteria** defined in Section 2.

1.5 Structure of the Paper

The paper is organised as follows:

- **Section 2** defines the formal correspondence requirements that must be satisfied for Einstein-domain dynamical equivalence.
- **Section 3** establishes the first-principles dynamics of the Φ substrate and its coupling to matter.
- **Section 4** introduces the coarse-graining framework required to obtain a continuum description.
- **Section 5** constructs the effective stress-energy tensor and demonstrates its conservation.
- **Section 6** defines the precise conditions under which Einstein-form closure is valid.

- **Section 7** derives the Einstein-domain dynamical equations and identifies the emergent gravitational coupling.
- **Section 8** characterises the correction tensor and its physical interpretation.
- **Section 9** demonstrates recovery of known gravitational limits.
- **Section 10** defines the domain of validity of the correspondence.
- **Section 11** identifies the conditions under which Einstein closure fails.
- **Section 12** provides a formal correspondence audit.
- **Section 13** clarifies the limits of the present claims.
- **Section 14** outlines the transition to empirical validation in Paper 4.3.
- **Section 15** summarises the conclusions.

1.6 Position Within the Correspondence Series

This work represents the second stage of the General Relativity correspondence within the GETT framework:

- **Paper 4.1:** emergence of spacetime geometry (kinematics) [29]
- **Paper 4.2 (this work):** emergence of Einstein-domain dynamics
- **Paper 4.3:** empirical validation against gravitational observables

Accordingly, this paper establishes the dynamical foundation required for testing GETT against the full suite of GR predictions.

1.7 Summary Statement

This paper demonstrates that the Einstein field equations arise as the leading-order dynamical closure of a coarse-grained, density-modulated Φ–matter system under explicitly defined physical conditions, thereby reconstructing General Relativity as a domain-limited effective theory grounded in underlying substrate dynamics.

2. Correspondence Standard for Dynamics

This section defines the formal requirements that must be satisfied for General Expanse Tension Theory (GETT) to achieve valid correspondence with the dynamical structure of General Relativity (GR). In Paper 4.1, an effective metric description was shown to emerge under defined physical conditions. Earlier emergent-gravity approaches, such as Sakharov's induced gravity, derive gravitational dynamics from quantum vacuum fluctuations without specifying an explicit microscopic coupling structure [14]. Similarly, thermodynamic derivations relate Einstein's equations to horizon entropy and local equilibrium conditions, without reference to an underlying field-theoretic Lagrangian [15]. However, the existence of a metric alone is not sufficient for GR correspondence. General Relativity is fundamentally a dynamical theory, in which spacetime geometry evolves in response to matter-energy content. Accordingly, correspondence requires that the effective metric arising in GETT:

- admits a well-defined curvature structure,
- couples consistently to an effective source, and
- evolves according to equations equivalent in form and predictive content to the Einstein field equations within a specified physical domain.

This section establishes the non-negotiable criteria against which all subsequent derivations in this paper are evaluated.

2.1 Structural Requirements

For valid dynamical correspondence, the emergent description must admit the full geometric and tensorial structure of GR [5-7]. Specifically, the effective metric $g_{\mu\nu}^{\text{eff}}$ must:

1. Support the construction of standard curvature objects:
   - Riemann curvature tensor $R^{\rho}{}_{\sigma\mu\nu}$,
   - Ricci tensor $R_{\mu\nu}$,
   - Ricci scalar R,
   - Einstein tensor $G_{\mu\nu}$.
2. Be compatible with a covariant derivative operator ∇_{μ} defined with respect to $g_{\mu\nu}^{\text{eff}}$.
3. Admit a well-defined causal and geometric structure consistent with the geometrisation limit established in Paper 4.1.
4. Support the formulation of local dynamical relations between curvature and source quantities.

These structures must arise internally from the effective description and must not be imposed independently of the underlying Φ-field dynamics.

2.2 Functional Requirement

Correspondence requires the existence of a dynamical relation between curvature and effective source content of the form:

$$G_{\mu\nu} = \mathcal{F}(T_{\mu\nu}^{\text{eff}})$$

where $\mathcal{F}$ is a mapping that reduces, in the Einstein domain, to a linear proportionality between curvature and stress-energy [1,2,5]. In the limit of valid correspondence, this relation must take the form:

$$G_{\mu\nu} = 8\pi G_{\text{eff}}\, T_{\mu\nu}^{\text{eff}} + \Delta_{\mu\nu}$$

where:

- $T_{\mu\nu}^{\text{eff}}$ is the effective stress-energy tensor derived from GETT substrate dynamics,
- G_{eff} is an emergent gravitational coupling constant, and
- $\Delta_{\mu\nu}$ represents higher-order or non-Einstein corrections.

The functional requirement is satisfied only if:

- the mapping between curvature and source is local and tensorial,
- the leading-order behaviour matches Einstein form, and
- all deviations are explicitly identifiable and controllable.

2.3 Conservation Requirement

A valid dynamical correspondence requires the existence of a locally conserved effective source. Specifically, the effective stress-energy tensor must satisfy:

$$\nabla_{\mu} T_{\mu\nu}^{\text{eff}} = 0$$

within the geometrisation regime [5,7].

In GETT, this conservation must arise as a consequence of the coarse-grained dynamics of the combined matter–Φ system, and not as an imposed postulate. Accordingly, the following must be demonstrated:

1. The total system admits local balance laws at the substrate level.
2. These balance laws persist under coarse-graining.
3. The resulting effective stress-energy tensor is conserved with respect to the emergent metric connection.

This requirement is essential: without conservation, no consistent curvature–source closure can be achieved.

2.4 Precision Requirement

Correspondence must hold to a level of precision consistent with all established observational tests of GR within the relevant domain. Accordingly:

1. The leading-order relation

$$G_{\mu\nu} \approx 8\pi G_{\text{eff}} \, T_{\mu\nu}^{\text{eff}}$$

   must reproduce GR predictions within observational tolerance.

2. The correction tensor $\Delta_{\mu\nu}$ must satisfy:
    - $|\Delta_{\mu\nu}| \ll |G_{\mu\nu}|$ in the Einstein domain,
    - be expressible in terms of identifiable physical quantities (e.g. gradients, coupling variation),
    - vanish or become negligible under defined limiting conditions.
3. The effective gravitational constant G_{eff} must be:
    - approximately constant within the domain of validity,
    - derivable from underlying coupling parameters,
    - stable under small perturbations of the system.

This requirement ensures that correspondence is not merely formal, but quantitatively valid.

2.5 Domain Qualification Requirement

Dynamical correspondence must be explicitly restricted to a well-defined physical regime. The theory must therefore:

1. Specify the conditions under which Einstein-form closure is valid.
2. Express these conditions as explicit inequalities involving:
    - coupling variation,
    - Φ-field gradients,
    - temporal evolution scales,
    - density regime.
3. Identify regimes in which correspondence fails, including:
    - low-density threshold domains,
    - regions of strong coupling modulation,
    - non-adiabatic or rapidly evolving systems,

- ultra-high-density regimes.

Correspondence is considered valid only within the domain where all required conditions are satisfied simultaneously.

2.6 Acceptance Criteria for Correspondence

GETT is considered to achieve valid Einstein-domain dynamical correspondence if, and only if, the following conditions are met:

- A consistent curvature framework is constructed from the emergent metric.
- A physically grounded effective stress-energy tensor is defined.
- Local conservation of the effective source is demonstrated.
- A curvature–source relation of Einstein form is derived to leading order.
- All correction terms are explicitly identified and shown to be negligible within the domain.
- The domain of validity is clearly defined and physically justified.

The formal requirements that must be satisfied for GETT to achieve valid Einstein-domain dynamical correspondence are summarised in Table 1 below.

Requirement Category	Formal Requirement	What Must Be Demonstrated	Failure Condition
Structural Requirement	Existence of full curvature framework	Effective metric supports Riemann, Ricci, scalar curvature, and Einstein tensor; covariant derivative well-defined	No consistent geometric/tensor structure from $g_{\mu\nu}^{\text{eff}}$
Functional Requirement	Curvature–source relation	Relation of form $G_{\mu\nu} = 8\pi G_{\text{eff}} T_{\mu\nu}^{\text{eff}} + \Delta_{\mu\nu}$ emerges	No local tensorial mapping between curvature and source
Conservation Requirement	Local conservation of effective source	$\nabla_\mu T_{\mu\nu}^{\text{eff}} = 0$ derived from substrate dynamics	No conserved effective stress-energy; inconsistent dynamics
Precision Requirement	Quantitative agreement with GR	Leading-order behaviour matches GR within observational tolerance; corrections small and controlled	Corrections large, uncontrolled, or unidentifiable
Correction Control Requirement *(implicit in precision)*	Explicit correction tensor	$\Delta_{\mu\nu}$ identified, parameterised, and negligible in Einstein domain	Corrections hidden, undefined, or dominant
Effective Coupling Requirement	Stable gravitational coupling	G_{eff} approximately constant and derivable from underlying parameters	Strong spatial/temporal variation in effective coupling
Domain Qualification Requirement	Explicit validity regime	Conditions for Einstein-domain validity defined via inequalities (density, gradients, time evolution)	No clear domain; applicability ambiguous
Breakdown Requirement	Explicit failure regimes	Clear identification of where and why GR correspondence fails	No defined breakdown → unfalsifiable or overclaimed

Table 1. Correspondence Requirements Grid

Table 1 shows the formal requirements that must be satisfied for GETT to achieve valid Einstein-domain dynamical correspondence. Each requirement defines a non-negotiable condition on structure, dynamics, conservation, precision, and domain validity. Failure to satisfy any requirement constitutes failure of correspondence.

Failure to satisfy any of these criteria constitutes a failure of dynamical correspondence. This specification establishes the standard against which all subsequent sections are evaluated.

The remainder of this paper demonstrates that, under explicitly defined physical conditions, the coarse-grained dynamics of the Φ-field satisfy these requirements, thereby recovering Einstein-domain gravitational dynamics as a leading-order effective description.

3. Substrate Dynamics of Φ (First-Principles Starting Point)

This section defines the fundamental dynamical framework of General Expanse Tension Theory (GETT) from which the effective gravitational description emerges [16-26]. The objective is to:

- specify the governing dynamics of the scalar substrate field Φ,
- define its energy–momentum content,
- establish its coupling to matter via the density-dependent modulation function $S(\Sigma)$, and
- restate gravity as an emergent phenomenon arising from Φ-field behaviour.

This section provides the complete physical starting point. All subsequent results – including the construction of the effective stress-energy tensor (Section 5) and the derivation of Einstein-domain dynamics (Section 7) – must follow from this foundation under the conditions defined in Section 6.

3.1 Field Dynamics

Scope: Define the governing equation of motion for the scalar field Φ, including its intrinsic dynamics, potential structure, and coupling to matter.

3.1.1 Fundamental Field Description

We consider a real, all-pervading gauge-invariant singlet scalar field $\Phi(x^\mu)$ defined over spacetime. The field represents the expanse substrate, which carries and mediates the tension associated with cosmological expansion and its interaction with matter. The dynamics of Φ are governed by a covariant field equation of Klein–Gordon type [9,10], with contributions from:

- intrinsic kinetic structure,
- a self-interaction potential $V(\Phi)$,
- and coupling to the matter sector via a density-dependent modulation function.

3.1.2 Governing Equation of Motion

The Φ field satisfies a generalized Klein–Gordon equation of the form:

$$\Box\Phi - \frac{dV}{d\Phi} = \mathcal{J}_{\text{int}}$$

where:

- $\Box \equiv \nabla^\mu \nabla_\mu$ is the covariant d'Alembertian operator,
- $V(\Phi)$ is the scalar potential,
- $\mathcal{J}_{\text{int}}$ represents the interaction source term arising from coupling to matter.

3.1.3 Interaction Source Term

The interaction term is governed by the density-dependent Higgs-portal-like coupling structure:

$$\mathcal{J}_{\text{int}} = \lambda_{\Phi H}(\Sigma)\, \mathcal{O}_{\text{matter}}$$

where:

- $\lambda_{\Phi H}(\Sigma) = \lambda_0\, S(\Sigma)$ is the effective coupling strength,
- $S(\Sigma)$ is the dimensionless modulation function depending on coarse-grained baryonic density Σ,
- $\mathcal{O}_{\text{matter}}$ represents the relevant matter-sector operator (e.g. proportional to $H^\dagger H$ or effective mass density).

This structure encodes the central GETT principle:

The interaction strength between the Φ field and matter is not constant but continuously modulated by local baryonic density.

3.1.4 Interpretation of Terms

The equation

$$\Box\Phi - \frac{dV}{d\Phi} = \mathcal{J}_{\text{int}}$$

has the following physical interpretation:

- $\Box\Phi$: propagation and redistribution of the Φ field,
- $\frac{dV}{d\Phi}$: intrinsic restoring or stabilizing structure of the field,
- $\mathcal{J}_{\text{int}}$: forcing term induced by the presence of matter.

In GETT, this interaction term reflects the fact that matter locally resists cosmological expansion, creating an imbalance in the Φ substrate and inducing tension gradients.

3.1.5 Relation to Section 6 Conditions

While the equation above is fully general, its role in Einstein-domain recovery depends on the conditions defined in Section 6.

In particular:

- Under **mid-density conditions (6.1)**,

  $\lambda_{\Phi H}(\Sigma) \approx \lambda_0 S_0$, enabling effective constant coupling.
- Under **weak coupling variation (6.2)**,

  spatial and temporal derivatives of $\lambda_{\Phi H}$are negligible at leading order.
- Under **smooth Φ-field conditions (6.3)**,

  higher-gradient terms remain perturbatively suppressed.
- Under **adiabatic evolution (6.4)**,

  rapid time-dependent forcing is absent.

3.1.6 Substrate Dynamics Grid

These conditions do not alter the fundamental equation but determine the regime in which it admits Einstein-domain closure. The fundamental components of the Φ-substrate dynamics and their roles in the GETT framework are summarised in Table 2 below.

Component	Definition / Structure	Physical Role	Dependency for Later Sections
Scalar Field Φ (Phi)	Real, gauge-invariant singlet scalar field defined over spacetime	Represents the expanse substrate carrying tension and mediating gravitational behaviour	Fundamental variable for all dynamics (Sections 4–7)
Field Equation	$\Box\Phi - \frac{dV}{d\Phi} = \mathcal{J}_{\text{int}}$	Governs propagation, self-interaction, and response of the Φ field to matter	Basis for coarse-graining (Section 4) and closure (Section 7)
Kinetic Term	$\Box\Phi = \nabla^\mu\nabla_\mu\Phi$	Describes redistribution and propagation of substrate tension	Determines gradient structure entering $\Delta_{\mu\nu}$
Potential $V(\Phi)$	Self-interaction potential of the scalar field	Provides intrinsic stability and restoring behaviour of the substrate	Contributes to $T^{\Phi}_{\mu\nu}$ (Section 3.2, 5)
Interaction Source $\mathcal{J}_{\text{int}}$	$\lambda_{\Phi H}(\Sigma)\,\mathcal{O}_{\text{matter}}$	Encodes forcing of Φ by matter (mass resisting expansion)	Drives emergence of effective source $T^{\text{eff}}_{\mu\nu}$
Coupling Function $\lambda_{\Phi H}(\Sigma)$	$\lambda_0 S(\Sigma)$	Density-modulated interaction strength between Φ and matter	Central to domain behaviour (Section 6) and G_{eff}
Modulation Function $S(\Sigma)$	Dimensionless function of baryonic density	Controls regime transitions and coupling variation	Governs Einstein-domain validity (Section 6)
Matter Operator $\mathcal{O}_{\text{matter}}$	Effective matter-sector source (e.g. Higgs-sector or density proxy)	Represents how matter sources Φ-field dynamics	Links to construction of $T^{\text{matter}}_{\mu\nu}$
Gravity (Reinterpreted)	Emergent from Φ tension gradients induced by matter	Restoring reaction force due to resistance to expansion	Connects substrate physics to effective curvature (Sections 4–7)
Energy–Momentum Content	Stress-energy tensor $T^{\Phi}_{\mu\nu}$ (defined in 3.2)	Encodes energy, pressure, and stress carried by Φ	Combined into $T^{\text{eff}}_{\mu\nu}$ (Section 5)
Density Field Σ	Coarse-grained baryonic density	Controls coupling modulation and regime transitions	Determines domain conditions (Section 6)

Table 2. Substrate Dynamics Grid

Table 2 shows the fundamental components of the Φ-substrate dynamics, including the governing field equation, coupling structure, and energy–momentum content, together with their physical roles and their dependencies in enabling coarse-graining, effective source construction, and Einstein-domain dynamical closure.

3.1.7 Role in the Overall Framework

This field equation is the **primary dynamical law** of GETT.

It governs:

- how the Φ substrate responds to matter,
- how tension gradients form,
- how energy and momentum are distributed within the field.

All emergent gravitational behaviour – including effective curvature and Einstein-domain dynamics – arises from this equation after coarse-graining and under the closure conditions established in Section 6. The Φ field obeys a covariant Klein–Gordon-type equation with a density-modulated coupling to matter, providing the fundamental dynamical mechanism from which emergent gravitational behaviour arises in GETT.

3.2 Energy–Momentum Content

Scope: Define the stress-energy tensor associated with the Φ field, establishing how energy, momentum, pressure, and stress are carried by the substrate and contribute to the effective gravitational source.

3.2.1 Definition from the Action

The energy–momentum content of the scalar field $\Phi$is defined through variation of the action with respect to the metric. Consider the Φ-sector action:

$$S_\Phi = \int d^4x \sqrt{-g}\left[-\frac{1}{2}\nabla^\mu\Phi\nabla_\mu\Phi - V(\Phi)\right]$$

The corresponding stress-energy tensor is given by:

$$T_{\mu\nu}^{\Phi} = \nabla_\mu\Phi\,\nabla_\nu\Phi - g_{\mu\nu}\left(\frac{1}{2}\nabla^\alpha\Phi\nabla_\alpha\Phi + V(\Phi)\right)$$

This is the standard symmetric, covariant stress-energy tensor for a real scalar field.

3.2.2 Physical Interpretation

The components of $T_{\mu\nu}^{\Phi}$ represent:

- **Energy density:** contributions from both kinetic and potential terms of the field,
- **Momentum density:** flow of the Φ field across spacetime,
- **Pressure and stress:** arising from gradients and field configuration.

In the GETT interpretation:

- $\nabla_\mu\Phi\nabla_\nu\Phi$ encodes **directional tension structure**,
- $V(\Phi)$ encodes **intrinsic energy stored in the substrate**,
- the full tensor represents the **distribution of expanse tension energy and stress**.

Thus, gravitational effects emerge from how this energy–momentum structure responds to matter-induced perturbations.

3.2.3 Inclusion of Interaction Contributions

The scalar field is coupled to matter via the density-dependent interaction term introduced in Section 3.1.

In a fully consistent treatment, the total energy–momentum content includes contributions from:

- the Φ field itself,
- the matter sector,
- and the interaction between them.

Accordingly, the complete effective source structure may be written schematically as:

$$T_{\mu\nu}^{\text{eff}} = T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^{\Phi} + T_{\mu\nu}^{\text{int}}$$

At the level of Section 3, we define explicitly only $T_{\mu\nu}^{\Phi}$. The interaction contribution $T_{\mu\nu}^{\text{int}}$ will be constructed and interpreted in Section 5 as part of the effective source.

3.2.4 Conservation Properties

The divergence of the Φ stress-energy tensor satisfies:

$$\nabla^{\mu} T_{\mu\nu}^{\Phi} = (\Box\Phi - \frac{dV}{d\Phi})\nabla_{\nu}\Phi$$

Using the field equation from Section 3.1,

$$\Box\Phi - \frac{dV}{d\Phi} = \mathcal{J}_{\text{int}}$$

this becomes:

$$\nabla^{\mu} T_{\mu\nu}^{\Phi} = \mathcal{J}_{\text{int}}\, \nabla_{\nu}\Phi$$

This shows that:

- the Φ sector alone is **not conserved** in the presence of coupling,
- energy–momentum is exchanged between Φ and matter via the interaction term.

Accordingly, only the combined system (matter + Φ + interaction) admits a conserved effective stress-energy tensor. This result is essential for Section 5, where conservation of $T_{\mu\nu}^{\text{eff}}$ will be demonstrated.

3.2.5 Role in Einstein-Domain Closure

The tensor $T_{\mu\nu}^{\Phi}$ provides one of the key contributions to the effective source entering the Einstein-domain dynamics. Under the conditions defined in Section 6:

- Φ-field gradients are smooth,
- coupling variation is weak,
- time evolution is adiabatic,

the contributions from $T_{\mu\nu}^{\Phi}$ can be consistently coarse-grained and combined with matter contributions to form a well-defined effective source $T_{\mu\nu}^{\text{eff}}$. In this regime, higher-order and non-universal contributions associated with the interaction sector are suppressed, enabling Einstein-form closure at leading order.

3.2.6 Interpretation in GETT

Within GETT, the stress-energy tensor of the Φ field represents the **physical carrier of gravitational dynamics**. Rather than curvature being fundamental, the structure encoded in $T_{\mu\nu}^{\Phi}$:

- reflects the distribution of tension and energy in the substrate,
- responds to matter-induced perturbations,
- and, after coarse graining, gives rise to an effective geometric description.

Thus, spacetime curvature in the Einstein domain is understood as the macroscopic manifestation of this underlying energy–momentum structure.

3.2.7 Summary Statement

The Φ field carries a well-defined, covariant stress-energy tensor derived from first principles, whose non-conservation in isolation reflects interaction with matter, and whose coarse-grained behaviour contributes directly to the effective source driving Einstein-domain gravitational dynamics.

3.3 Coupling to Matter (Higgs Portal $S(\Sigma)$)

Scope: Define the mechanism by which the Φ field couples to the matter sector, establishing the density-dependent modulation function $S(\Sigma)$ as the control parameter governing interaction strength and regime behaviour.

3.3.1 Interaction Structure

The coupling between the Φ field and the matter sector is implemented via a Higgs-portal-type interaction [11,12], in which the effective coupling strength is modulated by the local coarse-grained baryonic density Σ. At the Lagrangian level, the interaction term takes the schematic form:

$$\mathcal{L}_{\text{int}} = -\lambda_{\Phi H}(\Sigma)\,\Phi^2 H^\dagger H$$

where:

- H is the Standard Model Higgs doublet,
- $\lambda_{\Phi H}(\Sigma)$ is the density-dependent coupling function.

This structure preserves:

- gauge invariance of the Standard Model,
- Lorentz covariance,
- renormalisable interaction form (dimension-4 operator).

3.3.2 Density-Dependent Coupling Function

The defining feature of GETT is that the coupling strength is not constant, but depends explicitly on baryonic density:

$$\lambda_{\Phi H}(\Sigma) = \lambda_0\, S(\Sigma)$$

where:

- λ_0 is a constant baseline coupling parameter,
- $S(\Sigma)$ is a dimensionless modulation function.

The function $S(\Sigma)$ encodes how the interaction between the Φ field and matter varies continuously with the local environment.

3.3.3 Physical Interpretation of $S(\Sigma)$

The modulation function $S(\Sigma)$ represents the degree to which matter locally perturbs the Φ substrate.

Physically:

- matter resists cosmological expansion,
- this resistance induces local imbalance in the Φ field,
- the coupling strength reflects how strongly this imbalance feeds back into the field dynamics.

Thus, $S(\Sigma)$ controls the strength of the restoring response associated with mass–expansion interaction.

3.3.4 Regime Behaviour

The function $S(\Sigma)$ defines distinct physical regimes:

- **Mid-density regime:**
  $S(\Sigma) \approx S_0$,
  coupling approximately constant → Einstein-domain behaviour (Section 6)
- **Low-density regime:**
  $S(\Sigma)$varies significantly → modified dynamics (dark-matter-like behaviour)
- **Ultra-high-density regime:**
  coupling may change structure or weaken → symmetry restoration behaviour

The Einstein domain corresponds specifically to regions where $S(\Sigma)$ lies on a locally flat plateau, such that:

$$\frac{dS}{d\Sigma} \approx 0$$

3.3.5 Distinction from Other Scalar Theories

It is important to distinguish this mechanism from other scalar-field models:

- In chameleon or symmetron theories, environmental dependence arises through the minimisation of an effective potential.
- In GETT, by contrast, the density dependence is introduced directly at the level of the interaction coupling:

$$\lambda_{\Phi H} \rightarrow \lambda_{\Phi H}(\Sigma)$$

This means:

- no separate screening mechanism is required,

- modulation is continuous rather than triggered,
- the density field acts as a direct control parameter in the dynamics.

3.3.6 Role in Energy–Momentum Exchange

The interaction term contributes to the exchange of energy and momentum between the Φ field and the matter sector. As shown in Section 3.2:

$$\nabla^{\mu} T^{\Phi}_{\mu\nu} = \mathcal{J}_{\text{int}} \nabla_{\nu} \Phi$$

which reflects the fact that the Φ field alone is not conserved and the coupling transfers energy–momentum between sectors. The modulation function $S(\Sigma)$ therefore directly governs the strength and structure of this exchange.

3.3.7 Role in Einstein-Domain Closure

The behaviour of $S(\Sigma)$is central to the closure conditions defined in Section 6. Specifically:

- **Section 6.1 (Mid-density condition):**
  $S(\Sigma) \approx \text{const}$
- **Section 6.2 (Weak variation):**
  $|\nabla S|$and $|\partial_t S|$are negligible

When these conditions are satisfied:

- coupling becomes effectively constant,
- interaction terms simplify,
- non-universal effects are suppressed,
- Einstein-form closure becomes possible.

3.3.8 Conceptual Role in GETT

The density-modulated coupling function is the **central control mechanism** of the theory. It determines:

- when GR behaviour is recovered,
- when deviations occur,
- how transitions between regimes arise.

Thus, $S(\Sigma)$ is not an auxiliary feature – it is the **primary regulator of gravitational behaviour across domains**.

3.3.9 Summary Statement

The coupling between the Φ field and matter is governed by a density-dependent modulation function $S(\Sigma)$, implemented at the Lagrangian level, which controls the strength of interaction, defines physical regimes, and enables Einstein-domain dynamics to emerge when coupling variation is locally negligible.

3.4 Gravity Restated in GETT

Scope: Reinterpret gravity as an emergent physical phenomenon arising from Φ-field tension dynamics induced by matter, replacing the assumption of fundamental spacetime curvature with a substrate-based causal mechanism.

3.4.1 Statement of Principle

In General Expanse Tension Theory (GETT), gravity is not taken to be a fundamental interaction mediated by spacetime curvature. Instead, it is defined as a **restoring reaction force** arising from the interaction between matter and the Φ substrate.

Specifically:

Gravity is the macroscopic manifestation of tension gradients in the Φ field, generated when matter locally resists cosmological expansion.

3.4.2 Physical Mechanism

The mechanism proceeds as follows:

1. The Φ field encodes the background expansion of the universe as a real, physical substrate.
2. Matter, possessing mass, resists this expansion locally.
3. This resistance creates a **local imbalance** in the flow or configuration of the Φ field.
4. The imbalance generates **tension gradients** within the substrate.
5. These tension gradients induce a **restoring response**, which manifests as an attractive interaction between masses.

Thus, gravitational attraction arises from the system's tendency to restore equilibrium in the Φ field.

3.4.3 Relation to the Field Equation

This mechanism is encoded dynamically in the field equation:

$$\Box\Phi - \frac{dV}{d\Phi} = \mathcal{J}_{\text{int}}$$

Here:

- the interaction term $\mathcal{J}_{\text{int}}$ represents the influence of matter on the Φ field,
- the resulting field configuration carries gradients and stresses,
- these gradients determine the effective forces experienced by matter.

Thus, gravitational behaviour is not imposed externally but arises directly from the response of the Φ field to matter.

3.4.4 Emergence of Effective Geometry

While gravity is defined in terms of Φ-field dynamics, the effective metric description established in Paper 4.1 emerges as a coarse-grained representation of this underlying physics.

In the Einstein domain:

- the detailed substrate dynamics are not directly observable,
- their effects can be encoded in an effective spacetime geometry,
- trajectories of matter can be described as geodesics of an emergent metric.

Accordingly:

- spacetime curvature is not fundamental,
- it is the geometric encoding of Φ-field behaviour under appropriate conditions.

3.4.5 Relationship to Stress–Energy

The stress-energy tensor $T_{\mu\nu}^{\Phi}$, defined in Section 3.2, encodes the distribution of tension and energy within the Φ field.

Through coarse-graining:

- this energy–momentum structure combines with matter contributions,
- forming an effective source $T_{\mu\nu}^{\text{eff}}$,
- which, in the Einstein domain, governs the evolution of the emergent metric.

Thus, what appears in GR as "curvature sourced by stress-energy" is, in GETT, the macroscopic expression of substrate tension dynamics.

3.4.6 Role of the Coupling Function

The strength and structure of the gravitational response depend on the modulation function $S(\Sigma)$ defined in Section 3.3.

- When $S(\Sigma)$ is approximately constant,
  the restoring response is uniform → Einstein-domain behaviour.
- When $S(\Sigma)$ varies significantly,
  the restoring response changes → deviations from GR.

Thus, the coupling function governs how strongly matter perturbs the Φ substrate and how the resulting tension gradients behave.

3.4.7 Domain Interpretation

This reinterpretation leads to a clear domain-based understanding:

- **Einstein domain:**
  smooth, weakly varying, adiabatic conditions → GR emerges
- **Non-Einstein domains:**
  strong gradients, coupling variation, or rapid evolution → modified dynamics

Accordingly, gravity is not universally geometric; it is geometric only within the regime where the Φ substrate admits Einstein-form closure. Outside the Einstein domain, where one or more closure conditions fail, the correction tensor $\Delta_{\mu\nu}$ becomes dynamically significant. Such regimes are expected to arise in environments characterised by low baryonic density, strong coupling variation, or non-adiabatic behaviour, including, for example, the outer regions of galaxies, cosmological voids, and ultra-high-density compact-object interiors. These cases are not analysed here but are identified as domains in which departures from Einstein dynamics are expected to occur.

3.4.8 Conceptual Summary

The conceptual shift introduced here is:

- **GR view:**
  mass curves spacetime, and curvature dictates motion
- **GETT view:**
  mass perturbs a real physical substrate, generating tension gradients,
  and spacetime curvature is the effective macroscopic description of that process

In GETT, gravity is a restoring reaction arising from Φ-field tension gradients induced by matter's resistance to cosmological expansion, with spacetime curvature emerging as a coarse-grained geometric representation of this underlying physical mechanism in the Einstein domain.

Layer	What Is Established	Core Output	Enables
Field Ontology	Existence of a real, covariant singlet scalar field Φ as the physical substrate	A concrete physical medium underlying gravitational phenomena	Moves gravity from abstract geometry → physical mechanism
Dynamical Law	Governing equation of motion for Φ including self-interaction and matter coupling	Complete first-principles dynamical system	Basis for all subsequent derivations (Sections 4–7)
Energy–Momentum Structure	Well-defined stress-energy tensor $T^{\Phi}_{\mu\nu}$ derived from the action	Physical accounting of energy, momentum, and stress in the substrate	Required for constructing $T^{\text{eff}}_{\mu\nu}$ (Section 5)
Interaction Mechanism	Density-dependent coupling via modulation function $S(\Sigma)$	Explicit, continuous control of interaction strength	Defines regimes and enables domain-dependent behaviour (Section 6)
Energy–Momentum Exchange	Non-conservation of Φ alone; conservation only for combined system	Clear transfer mechanism between matter and substrate	Enables conserved effective source in Einstein domain
Physical Definition of Gravity	Gravity as a restoring reaction from Φ tension gradients	Concrete causal mechanism replacing abstract curvature assumption	Provides physical interpretation of emergent geometry
Emergent Geometry Link	Spacetime curvature as coarse-grained description of Φ dynamics	Geometry becomes an effective representation, not a primitive	Bridges directly to Section 4 (coarse-graining) and Section 7 (closure)
Domain Readiness	Framework compatible with closure conditions defined in Section 6	Substrate dynamics structured for controlled approximation	Ensures Einstein-domain recovery is derivable, not imposed

Table 3. Substrate Framework Grid (High-Level)

Table 3 shows the high-level structure of the Φ-substrate framework established in Section 3, summarising the physical foundations, dynamical laws, and interaction mechanisms from which Einstein-domain gravitational behaviour will be derived.

3.5 Constraints on the Scalar Potential $V(\Phi)$

The correspondence developed in this work does not depend on a specific functional form of the scalar potential $V(\Phi)$, but it does require that the potential satisfy minimal regularity and scale-separation conditions within the Einstein domain. Specifically, the following properties are required:

(A) Smoothness on Coarse-Graining Scales

The potential must vary slowly over the coarse-graining scales $L$and τ, such that

$$| \frac{V'(\Phi)}{V(\Phi)} | L \ll 1, | \frac{V''(\Phi)}{V(\Phi)} | L^2 \ll 1$$

within the domain of interest. This ensures that contributions from the potential do not introduce large local variations that would violate the smooth-field conditions required for Einstein closure.

(B) Stable Local Configuration

The Φ field must admit a locally stable configuration in the Einstein domain, characterised by

$$V'(\Phi_0) \approx 0, \qquad V''(\Phi_0) = m_\Phi^2 > 0$$

so that fluctuations $\delta\Phi$ are well-defined and governed by an effective mass scale m_Φ.

(C) Compatibility with Mid-Density Coupling Plateau

The potential must be compatible with the regime in which the coupling modulation satisfies

$$S(\Sigma) \approx S_0, \qquad \frac{dS}{d\Sigma} \approx 0$$

In particular, the combined Φ–matter system must admit a configuration in which both the potential and the coupling function are simultaneously slowly varying, ensuring that no competing gradients destabilise the Einstein-domain regime.

(D) Absence of Rapid Localised Features

The potential must not contain sharp features or rapid transitions on scales comparable to or smaller than the coarse-graining scale, i.e.

$$\ell_{\text{feature}} \gg L$$

so that coarse-grained quantities remain well-defined and averaging residuals remain perturbatively controlled.

3.5.1 Interpretation

These conditions are minimal and generic: they do not require fine-tuning of the potential but rather exclude pathological cases in which rapid variation or instability would invalidate the coarse-grained description.

Accordingly, the existence of the Einstein domain is not contingent on a specific choice of $V(\Phi)$, but follows for a broad class of smooth, stable scalar potentials compatible with the required scale separation.

3.5.2 Summary Statement

The scalar potential $V(\Phi)$ is required only to be smooth, locally stable, and compatible with the mid-density coupling plateau on coarse-graining scales, ensuring that Einstein-domain dynamics arise generically rather than through fine-tuned potential structure.

4. Coarse-Graining and Effective Description

This section establishes the formal procedure by which the microscopic dynamics of the Φ substrate, defined in Section 3, are mapped onto an effective macroscopic description compatible with General Relativity. The fundamental Φ-field dynamics describe a detailed, physically real substrate, including:

- local gradients and tension structure,
- density-dependent coupling to matter,
- energy–momentum exchange between sectors.

However, General Relativity operates at a coarse-grained, continuum level, in which:

- fine-scale substrate structure is not resolved,
- dynamics are expressed in terms of smooth fields,
- gravitational behaviour is encoded geometrically.

Accordingly, a systematic procedure is required to connect these two levels of description [4,5].

4.1 Objective

The objective of this section is to:

- define the coarse-graining process applied to the Φ–matter system,
- identify the conditions under which a continuum description is valid,
- construct the effective macroscopic variables that result from averaging,
- and establish the regime in which the emergent description can support Einstein-domain dynamics.

This section therefore provides the essential bridge between:

- **Section 3:** fundamental substrate dynamics,
- **Section 5:** effective stress-energy construction,
- **Section 7:** Einstein-domain dynamical closure.

4.2 Conceptual Framework

The coarse-graining procedure is based on the following principles:

1. **Separation of Scales**
   The Φ substrate may exhibit structure on scales smaller than those probed by macroscopic

gravitational phenomena. A continuum description is valid when these scales are well separated.

2. **Averaging Over a Finite Domain**
   Physical quantities are averaged over spacetime regions characterised by a spatial scale L and a temporal scale τ, as introduced in Section 6.
3. **Suppression of Fine-Scale Structure**
   Rapid fluctuations, sharp gradients, and non-adiabatic features that occur below the coarse-graining scale do not contribute at leading order to the effective description.
4. **Emergence of Smooth Fields**
   The result of coarse-graining is a set of smooth, effective variables – including energy density, pressure, and stress – that can be used to define a continuum theory.

4.3 Role in the Overall Framework

This section performs a critical function in the GETT correspondence programme.

Without coarse-graining:

- the Φ-field dynamics remain too detailed to admit a GR-like description,
- no effective stress-energy tensor can be consistently defined,
- no closed curvature–source relation can emerge.

With coarse-graining:

- the microscopic substrate is mapped to macroscopic continuum variables,
- a well-defined effective source $T_{\mu\nu}^{\text{eff}}$ can be constructed,
- and the conditions for Einstein-domain closure (Section 6) can be applied.

Thus, coarse-graining is not an auxiliary step – it is the mechanism by which geometry emerges from physics.

4.4 Key Requirement

The coarse-graining procedure must satisfy two essential criteria:

1. **Physical Consistency**
   It must preserve the underlying conservation laws and dynamical structure of the Φ–matter system.
2. **Closure Compatibility**
   It must produce effective variables that admit Einstein-form dynamical closure under the conditions defined in Section 6.

Coarse-graining provides the formal mapping from the detailed Φ-substrate dynamics to an effective continuum description, enabling the construction of macroscopic variables and the emergence of Einstein-domain gravitational behaviour under controlled physical conditions. Failure to satisfy either condition would prevent valid correspondence with GR.

4.5 Necessity of Coarse-Graining

Scope: Demonstrate that a direct mapping from Φ-substrate dynamics to Einstein-form gravitational behaviour is not possible without averaging, and that coarse-graining is a necessary step for the emergence of a continuum geometric description.

4.5.1 Incompatibility of Microscopic Dynamics with GR Form

The Φ-substrate dynamics defined in Section 3 describe a physically real field with:

- explicit spatial gradients and local structure,
- density-dependent coupling variation,
- interaction-driven energy–momentum exchange,
- potentially non-uniform and time-dependent behaviour.

In contrast, General Relativity is formulated in terms of:

- smooth metric fields,
- locally defined curvature tensors,
- a single effective stress-energy source,
- and closed dynamical equations of Einstein form.

These two descriptions operate at fundamentally different levels.

At the microscopic level:

- the field contains detailed structure not representable by a single smooth metric,
- coupling may vary across space and time,
- energy–momentum is not separately conserved within each sector.

Accordingly, no direct identification of Φ-field dynamics with Einstein equations is possible without further transformation.

4.5.2 Requirement for a Continuum Description

To recover GR-like behaviour, the description must be recast in terms of effective continuum variables. This requires:

- suppression of fine-scale structure,
- representation of dynamics by averaged quantities,
- elimination of explicit dependence on microscopic degrees of freedom.

Only under such a continuum description can:

- a well-defined stress-energy tensor be constructed,
- a consistent curvature–source relation be formulated,
- and a closed system of macroscopic equations be obtained.

Thus, coarse-graining is required to bridge the gap between microscopic substrate physics and macroscopic gravitational theory.

4.5.3 Role of Scale Separation

The validity of coarse-graining depends on the existence of a separation between:

- microscopic scales associated with Φ-field structure and interaction, and
- macroscopic scales associated with gravitational phenomena.

When such separation exists:

- variations at small scales average out,
- only the leading-order behaviour survives,
- the system can be described by smooth fields.

This condition is consistent with the closure requirements defined in Section 6, where gradients and variations are constrained to be small over the averaging domain.

4.5.4 Suppression of Non-Einstein Contributions

At the microscopic level, the Φ–matter system contains terms that are not present in Einstein's theory, including:

- higher-order derivative terms,
- coupling-gradient contributions,
- non-adiabatic temporal effects,
- non-universal interaction structure.

Coarse-graining suppresses these contributions by:

- averaging over fluctuations,
- reducing sensitivity to local variations,
- isolating the leading-order behaviour of the system.

The residual effects of these suppressed contributions are captured by the correction tensor $\Delta_{\mu\nu}$, introduced in Section 6.

4.5.5 Emergence of Effective Geometry

Once coarse-graining is applied:

- the detailed Φ-field structure is no longer directly resolved,
- its effects can be encoded in an effective metric $g_{\mu\nu}^{\text{eff}}$,
- and gravitational dynamics can be expressed geometrically.

Thus, geometry is not fundamental but emerges as the macroscopic encoding of averaged substrate dynamics. This establishes the necessary pathway:

Φ-substrate dynamics ⟶ coarse-grained variables ⟶ effective geometry.

4.5.6 Necessity Rather Than Choice

Coarse-graining is not introduced as a modelling convenience. It is required because:

- GR is inherently a continuum theory,
- the Φ-field description is inherently microscopic and structured,
- and no direct equivalence exists between the two without scale transformation.

Accordingly:

Einstein-domain dynamics cannot be derived directly from the microscopic Φ equations; they arise only after systematic coarse-graining under the conditions defined in Section 6.

4.5.7 Summary Statement

Coarse-graining is a necessary step in GETT, transforming the detailed, structured Φ-substrate dynamics into a smooth continuum description in which effective stress-energy, emergent geometry, and Einstein-domain gravitational behaviour can be consistently defined.

4.6 Averaging Procedure

Scope: Define the formal coarse-graining operation applied to the Φ–matter system, specifying the averaging scales and operators that produce the effective continuum variables.

4.6.1 Definition of Averaging Domain

Let $\mathcal{D}(x)$ denote a spacetime averaging region centred at a point x^μ, characterised by:

- a spatial scale L,
- a temporal scale τ.

The domain $\mathcal{D}(x)$ defines the resolution at which the effective continuum description is constructed. All microscopic quantities are averaged over this region to produce macroscopic variables.

4.6.2 Averaging Operator

For any scalar quantity $Q(x^\mu)$, define the coarse-grained (averaged) value as:

$$\langle Q\rangle(x) = \frac{1}{V_{\mathcal{D}}}\int_{\mathcal{D}(x)} Q(x')\, d^4x'$$

where:

- $V_{\mathcal{D}}$ is the spacetime volume of the averaging domain,
- x' spans the region $\mathcal{D}(x)$.

For tensorial quantities, the averaging is defined in a covariant manner consistent with the underlying geometry, ensuring that tensor character is preserved under the operation.

4.6.3 Averaging of the Φ Field

Applying the averaging operator to the scalar field yields:

$$\Phi_{\text{eff}}(x) = \langle\Phi\rangle(x),$$

which defines the effective substrate field entering the continuum description. Under the smooth-field conditions defined in Section 6:

- fluctuations of Φ within $\mathcal{D}$ are small,
- Φ_{eff} captures the dominant field behaviour,
- higher-order variations are suppressed.

4.6.4 Averaging of Energy–Momentum

The effective stress-energy tensor is defined as the averaged contribution of all sectors:

$$T_{\mu\nu}^{\text{eff}}(x) = \langle T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^{\Phi} + T_{\mu\nu}^{\text{int}} \rangle$$

This defines the macroscopic source entering the effective gravitational dynamics. The validity of this construction depends on:

- the suppression of fine-scale fluctuations,
- the existence of approximate locality at the coarse-grained level,
- and the conservation properties discussed in Sections 3.2 and 5.

4.6.5 Commutation with Derivatives

In general, averaging and differentiation do not commute:

$$\langle \nabla_\mu Q \rangle \neq \nabla_\mu \langle Q \rangle$$

The difference between these operations introduces correction terms associated with:

- gradients within the averaging domain,
- non-uniformities in the field,
- and higher-order structure.

Under the conditions defined in Section 6:

- these non-commuting contributions are suppressed,
- and may be consistently incorporated into the correction tensor $\Delta_{\mu\nu}$.

This is a key step in enabling Einstein-form closure at leading order.

4.6.6 Preservation of Conservation Laws

For the coarse-grained description to be physically valid, the averaging procedure must preserve conservation at the effective level. Specifically, if the full system satisfies local balance laws, then under appropriate conditions:

$$\nabla^\mu T_{\mu\nu}^{\text{eff}} \approx 0$$

Any residual violation arising from non-commutativity or unresolved structure must be subleading and included in $\Delta_{\mu\nu}$.

4.6.7 Validity Conditions

The averaging procedure is valid only when:

- the scales L and τ are large compared to microscopic structure,
- the conditions of Section 6 are satisfied,
- fluctuations within $\mathcal{D}$ are sufficiently small,
- and the system admits a well-defined continuum limit.

If these conditions fail:

- averaging becomes ill-defined,
- effective variables lose physical meaning,
- and Einstein-domain closure cannot be achieved.

4.6.8 Summary Statement

The averaging procedure defines a covariant coarse-graining operation over spacetime regions characterised by scales L and τ, producing effective fields and stress-energy tensors whose leading-order behaviour enables a continuum description and supports Einstein-domain gravitational dynamics.

4.7 Regime Conditions for Valid Averaging

Scope: Specify the physical and mathematical conditions under which the coarse-graining procedure defined in Section 4.6 yields a valid continuum description, consistent with the Einstein-domain closure conditions established in Section 6.

4.7.1 Requirement for Controlled Averaging

The averaging procedure introduced in Section 4.6 is only meaningful if the underlying Φ–matter system satisfies conditions that ensure:

- suppression of fine-scale structure,
- stability of averaged quantities,
- and consistency of the continuum approximation.

These requirements are not independent; they correspond directly to the closure conditions defined in Section 6.

4.7.2 Mapping to Closure Conditions

The validity of coarse-graining is guaranteed when the hypotheses (H1)–(H5) from Section 6 are satisfied.

Specifically:

- **Mid-density regime (6.1):**
  Ensures coupling remains approximately constant across the averaging domain.
- **Weak coupling variation (6.2):**
  Ensures that spatial and temporal variations in coupling do not introduce leading-order corrections.

- **Smooth Φ-field (6.3):**
  Ensures that field gradients and higher derivatives are small over the scale L.
- **Adiabatic evolution (6.4):**
  Ensures that temporal changes are slow relative to the averaging scale τ.
- **Universality (6.5):**
  Ensures that matter response is effectively uniform, allowing a single effective source description.

These conditions collectively ensure that averaging produces well-defined, slowly varying macroscopic fields.

4.7.3 Scale Consistency Conditions

For coarse-graining to be valid, the averaging scales must satisfy:

$$\ell_{\text{micro}} \ll L \ll \ell_{\text{macro}}, t_{\text{micro}} \ll \tau \ll t_{\text{macro}},$$

where:

- $\ell_{\text{micro}}, t_{\text{micro}}$: characteristic scales of microscopic Φ-field structure and interaction,
- $\ell_{\text{macro}}, t_{\text{macro}}$: characteristic scales of macroscopic gravitational phenomena.

This separation ensures that:

- microscopic fluctuations are averaged out,
- macroscopic variation is preserved,
- and the continuum approximation is meaningful.

4.7.4 Stability of Averaged Quantities

The averaged fields must vary slowly across neighbouring domains:

$$|\frac{\nabla\langle Q\rangle}{\langle Q\rangle}| L \ll 1, |\frac{\partial_t\langle Q\rangle}{\langle Q\rangle}| \tau \ll 1$$

for all relevant quantities Q, including:

- Φ,
- $S(\Sigma)$,
- and components of $T_{\mu\nu}$.

This ensures that the coarse-grained variables themselves form a smooth continuum field, consistent with GR.

4.7.5 Suppression of Averaging Residuals

As noted in Section 4.6, averaging and differentiation do not commute. The residual terms generated by this non-commutativity must satisfy:

$$\text{Residual terms} = O(\epsilon), \epsilon \ll 1$$

where ϵ is the small parameter defined in Section 6.

This ensures that:

- non-local or higher-order contributions remain subleading,
- and can be consistently absorbed into the correction tensor $\Delta_{\mu\nu}$.

4.7.6 Consistency with Conservation

The averaging procedure must preserve effective conservation laws. Under the above conditions:

- energy–momentum exchange between sectors averages consistently,
- the combined system admits an effective conserved tensor,
- and the condition

$$\nabla^\mu T_{\mu\nu}^{\mathrm{eff}} \approx 0$$

  holds to leading order.

This is essential for enabling Einstein-form closure in Section 7.

4.7.7 Failure of Valid Averaging

The coarse-graining procedure fails when the conditions above are violated.

This occurs when:

- coupling varies significantly across $\mathcal{D}$,
- Φ-field gradients are large,
- time evolution is non-adiabatic,
- universality breaks down,
- or scale separation does not exist.

In such regimes:

- averaged quantities become ill-defined,
- residual terms become leading order,
- and no consistent continuum description exists.

Accordingly, Einstein-domain dynamics cannot be recovered.

4.7.8 Summary Statement

The coarse-graining procedure yields a valid continuum description only when the system satisfies the Einstein-domain closure conditions, exhibits clear scale separation, and suppresses residual terms to subleading order, ensuring that effective variables are smooth, conserved, and compatible with Einstein-form dynamics.

4.8 Explicit Averaging Kernel and Residual Control

4.8.1 Definition of Averaging Operator

Introduce a clean, standard kernel:

$$\langle A(x)\rangle = \int d^4x'\, W_L(x-x')\, A(x')$$

where:

- $W_L(x)$ is a normalised smoothing kernel:

$$\int d^4x\, W_L(x) = 1$$

- characterised by a coarse-graining scale L (spatial) and τ (temporal).

Gaussian kernel (example):

$$W_L(x) \propto \exp\left(-\frac{|\mathbf{x}|^2}{2L^2} - \frac{t^2}{2\tau^2}\right)$$

4.8.2 Non-Commutation of Averaging and Derivatives

Stating explicitly:

$$\langle \nabla_\mu A\rangle \neq \nabla_\mu \langle A\rangle$$

Defining the residual:

$$R_\mu[A] = \langle \nabla_\mu A\rangle - \nabla_\mu \langle A\rangle$$

4.8.3 Scaling of Residuals (Critical Result)

Using standard smoothing estimates:

$$R_\mu[A] = O\left(\frac{\ell_{\text{micro}}}{L}\right)\ \partial_\mu A,$$

and similarly:

$$\langle \nabla\nabla A\rangle - \nabla\nabla\langle A\rangle = O\left(\frac{\ell_{\text{micro}}}{L}\right)$$

This scaling is independent of the specific choice of smoothing kernel, provided the kernel is normalised and characterised by a single coarse-graining scale L, so that all admissible kernels yield residuals controlled by the same dimensionless ratio ℓ_{micro}/L. Accordingly, kernel dependence enters only at higher order and is absorbed into the correction tensor $\Delta_{\mu\nu}$, ensuring that the Einstein-domain closure is insensitive to the detailed form of the averaging procedure at leading order.

4.8.4 Application to Φ Field

Applying directly to the system:

$$\langle \nabla_\mu \Phi \rangle = \nabla_\mu \langle \Phi \rangle + O\left(\frac{\ell_{\text{micro}}}{L}\right)$$

$$\langle \nabla_\mu \nabla_\nu \Phi \rangle = \nabla_\mu \nabla_\nu \langle \Phi \rangle + O\left(\frac{\ell_{\text{micro}}}{L}\right)$$

4.8.5 Closure Justification

Linking explicitly to Einstein closure, the small parameter governing coarse-graining validity is therefore:

$$\frac{\ell_{\text{micro}}}{L} \ll 1$$

which ensures that non-commutation residuals are perturbatively suppressed and can be consistently absorbed into the correction tensor $\Delta_{\mu\nu}$.

4.8.6 Interpretation

The averaging procedure therefore introduces no uncontrolled ambiguity; all deviations from exact commutation are explicitly parameterised and appear as higher-order contributions in the effective dynamics. A more detailed functional analysis of the coarse-graining procedure and kernel dependence will be presented in a dedicated technical treatment.

4.9 Emergent Continuum Variables

Scope: Define the effective macroscopic variables that arise from coarse-graining the Φ–matter system, establishing the continuum quantities that will enter the effective stress-energy tensor and support Einstein-domain dynamics.

4.9.1 Definition of Effective Fields

Under the averaging procedure defined in Section 4.6, the microscopic fields are mapped to smooth continuum variables.

In particular:

- the scalar field becomes

$$\Phi_{\text{eff}}(x) = \langle \Phi \rangle(x)$$

- the coupling function becomes

$$S_{\text{eff}}(x) = \langle S(\Sigma) \rangle(x)$$

- and all dynamical quantities are expressed in terms of their coarse-grained values.

These effective fields represent the macroscopic state of the system at the scale defined by L and τ.

4.9.2 Effective Energy Density and Pressure

The coarse-grained energy–momentum content can be decomposed into standard continuum variables. Defining:

- **Effective energy density**

$$\rho_{\text{eff}} = \langle T_{\mu\nu}^{\text{eff}} u^{\mu} u^{\nu} \rangle$$

- **Effective pressure and stress components**
  obtained from spatial projections of $T_{\mu\nu}^{\text{eff}}$,

where u^{μ} is the local four-velocity of the coarse-grained matter flow. These quantities describe how energy and momentum are distributed at the macroscopic level.

4.9.3 Decomposition of the Effective Stress-Energy Tensor

The effective stress-energy tensor can be expressed in standard fluid-like form:

$$T_{\mu\nu}^{\text{eff}} = \rho_{\text{eff}}\, u_{\mu} u_{\nu} + p_{\text{eff}}\, h_{\mu\nu} + \pi_{\mu\nu}$$

where:

- u^{μ} is the four-velocity field,
- $h_{\mu\nu} = g_{\mu\nu} + u_{\mu} u_{\nu}$ is the projection tensor,
- p_{eff} is the effective isotropic pressure,
- $\pi_{\mu\nu}$ represents anisotropic stress contributions.

This decomposition provides a direct bridge to the standard form used in GR.

4.9.4 Contributions from Φ and Matter

The effective quantities include contributions from:

- **Matter sector:**
  standard mass–energy content,
- **Φ field:**
  kinetic and potential energy, as well as tension structure,
- **Interaction sector:**
  energy–momentum exchange arising from the coupling $S(\Sigma)$.

Thus:

$$\rho_{\text{eff}} = \rho_{\text{matter}} + \rho_{\Phi} + \rho_{\text{int}}$$

with analogous decomposition for pressure and stress.

4.9.5 Behaviour in the Einstein Domain

Under the closure conditions defined in Section 6:

- $S(\Sigma) \approx$ const,
- gradients and time variations are small,
- anisotropic and higher-order contributions are suppressed,

so that:

- $\pi_{\mu\nu}$ becomes negligible at leading order,
- the effective stress-energy tensor approaches a standard perfect-fluid-like form,
- and the continuum variables behave smoothly.

This is the regime in which Einstein-form dynamics can emerge.

4.9.6 Residual Contributions

Deviations from ideal continuum behaviour arise from:

- higher-order Φ-field gradients,
- coupling variation across the averaging domain,
- non-adiabatic effects,
- non-universal matter response.

These contributions:

- are subleading under Section 6 conditions,
- are not discarded,
- and are systematically captured in the correction tensor $\Delta_{\mu\nu}$.

4.9.7 Role in the Overall Framework

The emergent continuum variables defined here provide:

- the inputs required for constructing $T_{\mu\nu}^{\text{eff}}$ in Section 5,
- the physical quantities that source the effective metric,
- and the bridge between substrate dynamics and geometric description.

They are the final step before dynamical closure.

Table 4, next page, shows the coarse-graining framework by which the microscopic Φ–matter dynamics are transformed into a continuum description, yielding effective variables, conservation structure, and geometric representation compatible with Einstein-domain gravitational dynamics.

Stage	What Is Performed	Key Output	Role in GR Correspondence
Necessity of Coarse-Graining (Sub-Section 4.5.6)	Establish that microscopic Φ dynamics cannot directly reproduce GR	Requirement for scale transformation	Justifies why GR must be an emergent, not fundamental, description
Averaging Procedure (Section 4.6)	Define covariant averaging over spacetime domain $\mathcal{D}(x)$ with scales L, τ	Coarse-grained fields and tensors	Provides formal mapping from microscopic to macroscopic variables
Regime Conditions (Section 4.7)	Apply closure conditions (Section 6) to ensure validity of averaging	Defined domain of applicability	Ensures continuum variables are well-behaved and physically meaningful
Scale Separation	Enforce $\ell_{\text{micro}} \ll L \ll \ell_{\text{macro}}$ and $t_{\text{micro}} \ll \tau \ll t_{\text{macro}}$	Suppression of microscopic fluctuations	Guarantees emergence of smooth continuum fields
Suppression of Residuals	Control non-commutativity of averaging and differentiation	Residual terms $O(\epsilon)$	Enables identification of correction tensor $\Delta_{\mu\nu}$
Emergent Fields (Section 4.9)	Define $\Phi_{\text{eff}}, S_{\text{eff}}$, and other averaged quantities	Smooth macroscopic field variables	Forms the basis of effective continuum description
Effective Stress-Energy Variables	Construct $\rho_{\text{eff}}, p_{\text{eff}}, \pi_{\mu\nu}$	Fluid-like decomposition of $T_{\mu\nu}^{\text{eff}}$	Directly feeds Einstein-domain source structure
Conservation Consistency	Ensure averaged system admits effective conservation	$\nabla^\mu T_{\mu\nu}^{\text{eff}} \approx 0$	Required for valid curvature–source dynamics
Emergent Geometry Link	Encode averaged dynamics into effective metric $g_{\mu\nu}^{\text{eff}}$	Geometric representation of physics	Enables transition to Einstein-form equations (Section 7)
Domain Readiness	Ensure compatibility with closure hypotheses (Section 6)	GR-compatible continuum regime	Completes bridge to Einstein-domain dynamical closure

Table 4. Coarse-Graining Framework for Emergent Continuum Description and Einstein-Domain Dynamics

4.9.8 Summary Statement

Coarse-graining of the Φ–matter system yields smooth continuum variables – including effective energy density, pressure, and stress – which combine into a well-defined effective stress-energy tensor that serves as the source of Einstein-domain gravitational dynamics.

5. Emergent Stress-Energy Tensor Construction

This section constructs the effective stress-energy tensor that governs the macroscopic gravitational dynamics in GETT. Building on:

- the substrate dynamics defined in Section 3, and
- the coarse-grained variables defined in Section 4,

we now define a single, conserved tensor $T_{\mu\nu}^{\text{eff}}$ that:

- encapsulates all relevant energy–momentum contributions,
- serves as the source of the effective metric, and
- enables Einstein-domain dynamical closure.

This construction is essential for satisfying the functional and conservation requirements defined in Section 2.

5.1 Effective Source Definition

Scope: Define the total effective stress-energy tensor as the coarse-grained combination of matter, Φ-field, and interaction contributions.

5.1.1 Total Effective Stress-Energy Tensor

The effective source governing gravitational dynamics is defined as:

$$T_{\mu\nu}^{\text{eff}} = \langle T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^{\Phi} + T_{\mu\nu}^{\text{int}} \rangle$$

where:

- $T_{\mu\nu}^{\text{matter}}$: standard matter-sector stress-energy tensor,
- $T_{\mu\nu}^{\Phi}$: scalar-field stress-energy tensor (Section 3.2),
- $T_{\mu\nu}^{\text{int}}$: interaction contribution arising from density-dependent coupling,
- $\langle \cdot \rangle$: coarse-graining operator defined in Section 4.6.

This defines a single macroscopic source tensor.

5.1.2 Physical Interpretation

The effective tensor represents the total energy–momentum content of the system at the continuum level:

- **Matter contribution:**
  mass–energy and momentum of baryonic matter,
- **Φ-field contribution:**
  energy stored in the substrate and its tension structure,
- **Interaction contribution:**
  energy–momentum exchange induced by coupling via $S(\Sigma)$.

Together, these form a closed physical system at the coarse-grained level.

5.1.3 Necessity of a Unified Source

A single effective tensor is required for consistency with GR correspondence. Einstein-form dynamics require:

- a single source $T_{\mu\nu}$,

- a single metric $g_{\mu\nu}$,
- and a single curvature–source relation.

If the contributions were not combined: multiple competing sources would exist, no unique curvature–source mapping could be defined, and Einstein-domain closure would fail. Thus, the construction of $T_{\mu\nu}^{\text{eff}}$ is a necessary step, not a choice.

5.1.4 Role of the Interaction Term

The interaction contribution $T_{\mu\nu}^{\text{int}}$ captures:

- energy–momentum transfer between matter and the Φ field,
- effects of the density-dependent coupling,
- non-trivial contributions arising from modulation function $S(\Sigma)$.

At the microscopic level, this term ensures neither matter nor Φ is conserved independently, only the total system admits conservation. At the coarse-grained level this contribution is incorporated into the unified effective tensor.

5.1.5 Behaviour in the Einstein Domain

Under the closure conditions defined in Section 6:

- coupling becomes approximately constant,
- interaction-induced non-universal effects are suppressed,
- higher-order contributions become negligible,

so that the effective tensor behaves as a smooth, well-defined continuum source, and the combined contributions can be treated as a single fluid-like entity. This is the regime in which Einstein-form dynamics can emerge.

5.1.6 Relation to Continuum Variables

Using the variables defined in Section 4.9, the effective tensor can be decomposed as:

$$T_{\mu\nu}^{\text{eff}} = \rho_{\text{eff}}\, u_\mu u_\nu + p_{\text{eff}}\, h_{\mu\nu} + \pi_{\mu\nu}$$

where:

- $\rho_{\text{eff}}, p_{\text{eff}}$: effective energy density and pressure,
- $\pi_{\mu\nu}$: anisotropic stress contributions.

This provides the direct connection to standard GR formulations.

5.1.7 Summary Statement

The effective stress-energy tensor $T_{\mu\nu}^{\text{eff}}$ is defined as the coarse-grained sum of matter, Φ-field, and interaction contributions, forming a single unified source that encodes all macroscopic energy–momentum content and enables Einstein-domain gravitational dynamics.

5.2 Interaction Stress–Energy Tensor

The effective stress–energy tensor introduced above contains a contribution arising from the interaction between the Φ field and the matter sector. At the level of the underlying action, this interaction is mediated through the Higgs portal term

$$\mathcal{L}_{\text{int}} = -\lambda_{\Phi H}(\Sigma)\, \Phi^2\, H^\dagger H$$

with

$$\lambda_{\Phi H}(\Sigma) = \lambda_0\, S(\Sigma)$$

5.2.1 Definition via Metric Variation

The interaction contribution to the stress–energy tensor is defined in the standard way by variation of the interaction Lagrangian with respect to the metric:

$$T_{\mu\nu}^{\text{int}} = -\frac{2}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g}\,\mathcal{L}_{\text{int}}\right)}{\delta g^{\mu\nu}}$$

This expression captures both:

- explicit dependence of $\mathcal{L}_{\text{int}}$ on the metric,
- and implicit dependence through the fields Φ, H, and the density Σ.

5.2.2 Schematic Form

To leading order, and suppressing higher-order derivative and curvature couplings, the interaction stress–energy tensor takes the schematic form

$$T_{\mu\nu}^{\text{int}} \sim \lambda_{\Phi H}(\Sigma)\, \Phi^2\, H^\dagger H\, g_{\mu\nu} + (\text{derivative and coupling-variation terms})$$

where additional contributions arise from:

- gradients of the fields $\nabla_\mu \Phi$, $\nabla_\mu H$,
- and spacetime variation of the coupling $\nabla_\mu \lambda_{\Phi H}(\Sigma)$.

5.2.3 Einstein-Domain Reduction

Following electroweak symmetry breaking, $H^\dagger H \to v^2 + \delta h$, and in the Einstein domain:

- $S(\Sigma) \approx S_0$,
- field gradients are small,
- and coupling variation is suppressed,

so that the leading contribution reduces to

$$T_{\mu\nu}^{\text{int}} \approx \lambda_0\, S_0\, v^2\, \Phi^2\, g_{\mu\nu} + O(\epsilon)$$

Thus, the interaction term contributes an effective isotropic energy density at leading order, with higher-order corrections entering through the same small parameters governing Einstein-domain closure.

5.2.4 Role in Effective Stress–Energy

The total effective stress–energy tensor may therefore be written schematically as

$$T_{\mu\nu}^{\text{eff}} = T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^{\Phi} + T_{\mu\nu}^{\text{int}}$$

with the interaction term encoding the coupling between the Φ field and the matter sector.

5.2.5 Scope and Interpretation

The expression above is schematic and sufficient for establishing:

- the existence and structure of the interaction contribution,
- its dependence on the coupling function $\lambda_{\Phi H}(\Sigma)$,
- and its role in the effective macroscopic dynamics.

A full evaluation of $T_{\mu\nu}^{\text{int}}$, including all derivative and higher-order contributions, is not required for the correspondence analysis and is deferred to future work.

5.3 Physical Interpretation of Components

Scope: Provide a clear physical interpretation of the individual contributions to the effective stress-energy tensor, clarifying how matter, the Φ field, and their interaction combine to produce the macroscopic gravitational source.

5.3.1 Decomposition of Contributions

The effective stress-energy tensor may be written as:

$$T_{\mu\nu}^{\text{eff}} = \langle T_{\mu\nu}^{\text{matter}} \rangle + \langle T_{\mu\nu}^{\Phi} \rangle + \langle T_{\mu\nu}^{\text{int}} \rangle$$

Each term represents a distinct physical component of the system, which together define the total energy–momentum content.

5.3.2 Matter Contribution

The matter term $T_{\mu\nu}^{\text{matter}}$ represents:

- baryonic mass–energy density,
- momentum flow associated with matter motion,
- pressure and stress within the matter sector.

In the context of GETT:

- this term reflects the source of resistance to expansion,
- it is the primary driver of perturbations in the Φ substrate,
- and it anchors the effective source in observable matter content.

5.3.3 Φ-Field Contribution

The scalar field contribution $T_{\mu\nu}^{\Phi}$ encodes:

- kinetic energy of the Φ field,
- potential energy stored in $V(\Phi)$,
- directional stress arising from field gradients.

Physically, this term represents:

- the energy and tension structure of the expanse substrate,
- the medium through which gravitational effects are mediated,
- the carrier of restoring forces generated by matter-induced imbalance.

This is the key departure from GR: part of what sources gravity is not matter alone, but the response of the substrate itself.

5.3.4 Interaction Contribution

The interaction term $T_{\mu\nu}^{\text{int}}$ represents:

- energy–momentum exchange between matter and the Φ field,
- effects of the density-dependent coupling $S(\Sigma)$,
- contributions arising from modulation of interaction strength.

This term is responsible for:

- transferring energy between sectors,
- encoding how strongly matter perturbs the substrate,
- introducing environment-dependent behaviour.

It is therefore the mechanism that links matter to Φ-field dynamics.

5.3.5 Combined System Behaviour

At the microscopic level:

- the three components are dynamically coupled,
- energy–momentum is exchanged between them,
- no individual component is independently conserved.

At the coarse-grained level:

- these contributions combine into a single effective tensor,
- the system behaves as a unified continuum source,
- conservation emerges at the level of the total tensor.

Thus, $T_{\mu\nu}^{\text{eff}}$ represents the collective behaviour of the coupled system, not a simple sum of independent parts.

5.3.6 Behaviour in the Einstein Domain

Under the conditions defined in Section 6:

- $S(\Sigma) \approx \text{const}$,
- coupling variation is negligible,
- gradients and time-dependence are suppressed,

so that:

- the interaction term becomes effectively uniform,
- non-universal contributions are negligible,
- the combined system behaves like a single effective fluid.

In this regime the distinction between components becomes less dynamically significant at leading order, and the effective tensor reduces to the standard form required for Einstein-domain dynamics.

5.3.7 Interpretation of Deviations

Outside the Einstein domain, the relative contributions of the three components become more pronounced:

- the Φ-field contribution may dominate in low-density regimes,
- the interaction term may introduce non-trivial corrections,
- the effective source may deviate from standard fluid behaviour.

These deviations are captured by the correction tensor $\Delta_{\mu\nu}$ introduced in Section 6.

5.3.8 Conceptual Summary

The effective stress-energy tensor in GETT is not merely a description of matter, but of a coupled system in which:

- matter provides the source of perturbation,
- the Φ field provides the physical medium and response,
- and their interaction governs the structure of gravitational dynamics.

5.3.9 Summary Statement

The effective stress-energy tensor represents the combined energy–momentum content of matter, the Φ substrate, and their interaction, whose unified coarse-grained behaviour defines the macroscopic source of gravitational dynamics in GETT.

5.4 Conservation Structure

Scope: Demonstrate that the coarse-grained effective stress-energy tensor is locally conserved, establishing the conservation requirement necessary for Einstein-domain dynamical closure.

5.4.1 Conservation at the Substrate Level

At the fundamental level, the Φ–matter system is governed by:

- the scalar field equation (Section 3.1),
- the matter-sector dynamics,
- and the interaction between them.

From Section 3.2, the divergence of the Φ-field stress-energy tensor is:

$$\nabla^\mu T^{\Phi}_{\mu\nu} = \mathcal{J}_{\text{int}}\, \nabla_\nu \Phi$$

Similarly, the matter-sector stress-energy tensor satisfies:

$$\nabla^\mu T^{\text{matter}}_{\mu\nu} = -\mathcal{J}_{\text{int}}\, \nabla_\nu \Phi$$

where the sign reflects the exchange of energy–momentum between matter and the Φ field [5,7].

Thus, at the microscopic level neither sector is conserved independently, and energy–momentum is transferred via the interaction term.

5.4.2 Conservation of the Total System

Combining the two contributions yields:

$$\nabla^\mu \left(T^{\text{matter}}_{\mu\nu} + T^{\Phi}_{\mu\nu} \right) = 0$$

When the interaction contribution $T^{\text{int}}_{\mu\nu}$ is included explicitly, the full system satisfies:

$$\nabla^\mu \left(T^{\text{matter}}_{\mu\nu} + T^{\Phi}_{\mu\nu} + T^{\text{int}}_{\mu\nu} \right) = 0$$

Thus, the total energy–momentum of the coupled system is conserved at the fundamental level.

5.4.3 Coarse-Grained Conservation

Applying the averaging operator defined in Section 4.6, we define:

$$T^{\text{eff}}_{\mu\nu} = \langle T^{\text{matter}}_{\mu\nu} + T^{\Phi}_{\mu\nu} + T^{\text{int}}_{\mu\nu} \rangle$$

Under the regime conditions specified in Sections 4.7 and 6:

- fluctuations within the averaging domain are small,
- residual terms arising from averaging are suppressed,
- scale separation is maintained.

Accordingly, the divergence of the effective tensor satisfies:

$$\nabla^\mu T^{\text{eff}}_{\mu\nu} = O(\epsilon), \qquad \epsilon \ll 1$$

This establishes a two-step conservation structure, in which exact microscopic balance is preserved under coarse-graining with explicitly controlled residuals, ensuring that effective stress–energy conservation holds within the Einstein domain.

5.4.4 Einstein-Domain Limit

In the Einstein domain, defined by the closure conditions (Section 6):

- coupling variation is negligible,
- Φ-field gradients are smooth,
- evolution is adiabatic,
- non-universal effects are suppressed,

so that:

$$\nabla^{\mu} T_{\mu\nu}^{\text{eff}} \approx 0$$

In the strict limit $\epsilon \to 0$, conservation holds exactly:

$$\nabla^{\mu} T_{\mu\nu}^{\text{eff}} = 0$$

This satisfies the **conservation requirement** defined in Section 2.3.

5.4.5 Role of Residual Terms

Any deviations from exact conservation arise from:

- non-commutativity of averaging and differentiation,
- coupling gradients,
- higher-order Φ-field structure,
- non-adiabatic dynamics.

These contributions are:

- explicitly controlled by the small parameter ϵ,
- subleading in the Einstein domain,
- and incorporated into the correction tensor $\Delta_{\mu\nu}$.

Thus, conservation is not assumed – it is recovered as a controlled limit.

5.4.6 Necessity for Einstein Closure

Local conservation of the effective stress-energy tensor is essential for:

- the existence of a consistent curvature–source relation,
- compatibility with the geometric framework of Section 4,
- and the derivation of Einstein-form equations in Section 7.

Without conservation:

- the system would not admit a closed set of gravitational field equations,
- and dynamical correspondence with GR would fail.

5.4.7 Conceptual Interpretation

In GETT, conservation is not imposed as a fundamental axiom. Instead, it arises from the internal consistency of the coupled Φ–matter system and becomes exact only in the regime where interaction effects are balanced and well-behaved. Thus, conservation is an emergent property of the Einstein domain, not a universal requirement across all regimes.

5.4.8 Summary Statement

The effective stress-energy tensor constructed from the coarse-grained Φ–matter system is locally conserved to leading order under the Einstein-domain conditions, with all deviations explicitly controlled and suppressed, thereby satisfying the conservation requirement necessary for Einstein-form dynamical closure.

Stage	What Is Established	Key Output	Role in GR Correspondence
Unified Source Definition (Section 5.1)	Combination of matter, Φ-field, and interaction contributions into a single tensor	$T_{\mu\nu}^{\text{eff}} = \langle T_{\mu\nu}^{\text{matter}} + T_{\mu\nu}^{\Phi} + T_{\mu\nu}^{\text{int}} \rangle$	Provides the single source required for Einstein-form dynamics
Physical Composition (Section 5.3)	Interpretation of each contributing sector	Matter (source), Φ (substrate), interaction (exchange)	Demonstrates full physical content behind the effective tensor
Energy–Momentum Exchange	Non-conservation of individual sectors; conservation of total system	Coupled system with internal energy transfer	Ensures physical consistency of the framework
Continuum Representation	Decomposition into fluid-like variables	$\rho_{\text{eff}}, p_{\text{eff}}, \pi_{\mu\nu}$	Aligns with standard GR stress-energy formulations
Einstein-Domain Behaviour	Suppression of coupling variation and higher-order terms	Smooth, approximately perfect-fluid behaviour	Enables leading-order Einstein closure
Conservation Structure (Section 5.4)	Local conservation of effective stress-energy tensor	$\nabla^{\mu} T_{\mu\nu}^{\text{eff}} \approx 0$	Satisfies core GR requirement for dynamical consistency
Residual Control	Identification and suppression of non-commuting and higher-order effects	Corrections $O(\epsilon)$	Feeds into correction tensor $\Delta_{\mu\nu}$
Domain Dependence	Validity tied to Section 6 conditions	Einstein domain explicitly defined	Prevents overreach; ensures falsifiability
Closure Readiness	Fully defined, conserved macroscopic source	GR-compatible $T_{\mu\nu}^{\text{eff}}$	Direct input to Einstein-form field equation (Section 7)

Table 5. Section 5 Summary — Effective Stress-Energy Construction Grid

Table 5 shows the construction of the effective stress-energy tensor, summarising how the coupled matter–Φ system is combined, interpreted, and shown to be conserved at the macroscopic level, thereby providing the unified source required for Einstein-domain gravitational dynamics.

6. Conditions for Einstein Closure (Theorem Hypotheses)

The closure conditions defined below may be expressed in a manifestly covariant form by constructing scalar invariants from covariant derivatives of the relevant fields.

6.1 Mid-Density Regime Condition

Scope: Require that the coupling between the Φ field and matter is effectively constant over the coarse-graining domain, establishing the regime in which Einstein-domain closure becomes possible.

Let u^μ denote the coarse-grained timelike 4-velocity of the matter flow, and define the spatial projection operator

$$h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$$

Spatial and temporal variations are then characterised by the projected derivative $h_{\mu\nu}\nabla_\mu$ and the convective derivative $u^\mu\nabla_\mu$, respectively. The closure conditions may thus be written in terms of dimensionless scalar ratios that are invariant under coordinate transformations.

6.1.1 Statement of Condition

Let $\Sigma(\mathbf{x}, t)$ denote the coarse-grained baryonic density field and let $S(\Sigma)$ denote the density-dependent coupling modulation function governing the interaction between the Φ field and the matter sector.

The **mid-density regime condition** is defined by:

$$S(\Sigma) \approx S_0$$

where S_0 is a constant, and deviations from constancy satisfy:

$$|\frac{S(\Sigma) - S_0}{S_0}| \ll 1$$

throughout the coarse-graining domain.

6.1.2 Physical Interpretation

This condition asserts that, within the Einstein domain:

- the coupling between the Φ field and matter is locally uniform,
- density-dependent modulation effects are inactive to leading order,
- the system resides away from any density thresholds or transition regions.

In this regime, the Φ–matter interaction behaves as if governed by a constant effective coupling, enabling a universal response of matter to Φ-field dynamics.

6.1.3 Role in Einstein Closure

The approximate constancy of $S(\Sigma)$ is a necessary condition for:

- the emergence of a stable effective gravitational coupling G_{eff},
- the suppression of non-universal interaction terms,

- the elimination (to leading order) of explicit density-gradient-driven source corrections.

Without this condition, the curvature–source relation cannot reduce to Einstein form, as spatial or temporal variation in coupling would introduce additional terms incompatible with GR dynamics.

6.1.4 Domain Qualification

This condition is satisfied in regions where:

- the baryonic density lies within the mid-density plateau of the coupling function,
- $\frac{dS}{d\Sigma} \approx 0$,
- the system is sufficiently far from identified GETT transition thresholds (e.g. low-density or ultra-high-density regimes).

6.1.5 Failure Mode

The condition breaks down when:

- Σ approaches a density threshold where $S(\Sigma)$ varies rapidly,
- $|\frac{dS}{d\Sigma}|$becomes significant,
- coupling modulation becomes dynamically relevant.

In such regimes:

- additional source terms arise,
- effective universality is lost,
- Einstein-domain closure fails.

6.1.6 Summary Statement

The mid-density regime condition ensures that coupling modulation is negligible, permitting the Φ–matter interaction to be treated as effectively constant and enabling the emergence of Einstein-domain dynamics.

6.2 Weak Coupling Variation Condition

Scope: Constrain the spatial and temporal variation of the coupling modulation function to be negligible across the coarse-graining domain, so that Einstein-form closure is not disrupted by gradient-driven interaction terms.

6.2.1 Statement of Condition

Let $S(\Sigma)$ denote the density-dependent coupling modulation function and let L and τ denote the characteristic spatial and temporal coarse-graining scales, respectively.

The weak coupling variation condition requires that the fractional variation of Sacross the averaging domain be small:

$$|\frac{\nabla_i S}{S}|L \ll 1$$

and, where temporal variation is relevant,

$$\left|\frac{\partial_t S}{S}\right| \tau \ll 1$$

Equivalently, the coupling function must remain approximately constant not only in value, but also in its local variation across the spacetime region over which effective continuum dynamics are defined.

6.2.2 Meaning of the Coarse-Graining Scales

Here:

- L is the characteristic spatial scale over which microscopic substrate structure is averaged to produce the effective continuum description;
- τ is the corresponding temporal averaging scale over which rapid microscopic response is smoothed.

These scales define the resolution at which Einstein-domain dynamics are claimed to emerge. The condition therefore states that coupling variation must be negligible at the level of the effective description, not necessarily at arbitrarily small, microscopic scales.

6.2.3 Physical Interpretation

This condition refines the mid-density requirement of Section 6.1.

Section 6.1 states that the system lies in a regime where $S(\Sigma) \approx S_0$.
Section 6.2 strengthens this by requiring that any residual variation in S is too small to generate leading-order dynamical corrections across the coarse-grained region.

Physically, this means the coupling landscape is locally flat, matter samples essentially the same coupling strength throughout the averaging volume, and no significant curvature-source correction arises from coupling gradients themselves. Thus, the effective gravitational response can be treated as locally stable and approximately uniform.

6.2.4 Role in Einstein Closure

This condition is required because gradients of $S(\Sigma)$ generically generate additional terms in the effective field equations.

If S varies appreciably across the averaging scale, then:

- interaction terms become position-dependent,
- source contributions cease to be locally universal,
- extra derivative terms appear in the effective dynamical closure,
- the Einstein-form relation

$$G_{\mu\nu} = 8\pi G_{\text{eff}} T_{\mu\nu}^{\text{eff}}$$

  no longer holds at leading order without correction.

By imposing weak coupling variation, these derivative-driven contributions are demoted to higher-order terms and absorbed into the correction tensor $\Delta_{\mu\nu}$.

6.2.5 Relation to the Mid-Density Plateau

In practice, this condition is naturally satisfied when the system lies on a sufficiently flat portion of the coupling function $S(\Sigma)$, such that:

$$S(\Sigma) \approx S_0, \qquad \left| \frac{dS}{d\Sigma} \right| \approx 0$$

throughout the relevant density interval.

Using the chain rule, spatial variation in S may be written schematically as

$$\nabla_i S = \frac{dS}{d\Sigma} \nabla_i \Sigma$$

Thus, even where density itself varies across the system, Einstein closure remains viable provided the coupling function is sufficiently flat that such density variation does not translate into appreciable coupling variation. This distinction is important: GR correspondence does not require density to be spatially constant, only that the coupling response to density be effectively constant over the domain.

6.2.6 Failure Mode

The weak coupling variation condition fails when either:

$$\frac{\sqrt{h^{\mu\nu}\nabla_\mu S \nabla_\nu S}}{| S |} L \sim 1 \qquad \text{or} \qquad \frac{| u^\mu \nabla_\mu S |}{| S |} \tau \sim 1$$

or

$$\left| \frac{\partial_t S}{S} \right| \tau \sim 1$$

This occurs near:

- active threshold regions,
- sharp density transitions,
- rapidly evolving environments in which the effective coupling changes materially over the averaging domain.

In such cases:

- derivative coupling terms contribute at leading order,
- G_{eff} cannot be treated as locally stable,
- non-Einstein corrections become dynamically significant.

Accordingly, Einstein-domain closure is no longer valid.

6.2.7 Summary Statement

The weak coupling variation condition requires that residual gradients in the modulation function $S(\Sigma)$ be negligible across the coarse-graining domain, ensuring that coupling-driven derivative terms remain subleading and that Einstein-form closure can be maintained to leading order.

6.3 Smooth Φ-Field Condition

Scope: Require that the coarse-grained Φ field vary sufficiently slowly across the averaging domain that higher-gradient substrate structure does not contribute at leading order to the effective gravitational dynamics.

6.3.1 Statement of Condition

Let $\Phi(\mathbf{x}, t)$ denote the coarse-grained scalar substrate field and let L and τ denote the characteristic spatial and temporal coarse-graining scales.

The smooth Φ-field condition requires that the normalized variation of the field across the averaging domain be small:

$$\frac{\sqrt{h^{\mu\nu}\,\nabla_\mu\Phi\,\nabla_\nu\Phi}}{|\,\Phi\,|}\,L \ll 1$$

$$\frac{\sqrt{h^{\mu\nu}h^{\alpha\beta}\,\nabla_\mu\nabla_\alpha\Phi\,\nabla_\nu\nabla_\beta\Phi}}{|\,\Phi\,|}\,L^2 \ll 1$$

and, where temporal variation is relevant,

$$|\,\frac{\partial_t\Phi}{\Phi}\,|\,\tau \ll 1$$

At the next level, Einstein closure further requires suppression of curvature-generating higher-gradient structure, so that second-derivative contributions remain perturbatively controlled. This may be expressed schematically as:

$$|\,\frac{\nabla_i\nabla_j\Phi}{\Phi}\,|\,L^2 \ll 1$$

with analogous temporal or mixed derivative suppression where required.

6.3.2 Physical Meaning

This condition states that the Φ substrate must be locally smooth on the scale at which the continuum description is defined. The existence of the Einstein domain also requires that the scalar potential $V(\Phi)$ be smooth and locally stable on the coarse-graining scale; see Section 3.5 for precise conditions. It does not require the Φ field to be spatially uniform or dynamically inactive. Rather, it requires that:

- field variation is gradual rather than abrupt,
- substrate structure does not fluctuate strongly within a coarse-graining cell,
- the emergent metric can be constructed from a slowly varying field background.

In physical terms, the medium may carry tension, gradients, and dynamical structure, but not in a way that produces rapid intra-domain variation incompatible with Einstein-form truncation.

6.3.3 Role in Einstein Closure

This condition is essential because the effective metric in GETT is ultimately sourced by the behaviour of the Φ field.

If Φ varies too sharply across the averaging scale, then:

- higher-derivative field terms remain dynamically important,
- anisotropic or non-local substrate structure can survive coarse-graining,
- additional non-Einstein contributions enter the curvature–source relation,
- no controlled leading-order truncation to Einstein form is possible.

By imposing smooth Φ variation, the dominant substrate contribution can be captured by the leading continuum variables, while residual higher-gradient effects are collected into the correction tensor $\Delta_{\mu\nu}$.

6.3.4 Why First and Second Derivatives Both Matter

The first-derivative conditions ensure that the field amplitude does not vary strongly across the averaging domain. However, Einstein-domain closure also depends on the suppression of strong local bending or rapid internal structure in the field configuration. For that reason, second-derivative control is also required. The distinction is important:

- small first derivatives imply slow variation,
- small second derivatives imply the absence of sharp local structure or rapidly changing gradients.

Together, these ensure that the coarse-grained field behaves as a smooth continuum substrate rather than as a strongly structured microscopic medium whose detailed dynamics would survive into the effective gravitational description.

6.3.5 Relationship to Coarse-Graining

The smooth Φ-field condition is defined relative to the chosen coarse-graining scales L and τ.

This means that the condition does not assert smoothness at arbitrarily small scales. Instead, it asserts that any microscopic fine structure is sufficiently subdominant that, after averaging, the effective field entering the dynamical closure is slowly varying. This is a standard requirement for any successful continuum limit: the field need not be exactly featureless, but unresolved short-scale structure must not contribute at leading order to the macroscopic evolution equations.

6.3.6 Failure Mode

The condition fails when the Φ field develops significant variation across the averaging scale, such that one or more of the quantities

$$\left|\frac{\nabla_i\Phi}{\Phi}\right| L, \left|\frac{\partial_t\Phi}{\Phi}\right| \tau, \left|\frac{\nabla_i\nabla_j\Phi}{\Phi}\right| L^2$$

cease to be small. This may occur in regimes involving:

- threshold crossing,
- strong local tension gradients,
- rapidly varying substrate response,
- highly structured or strongly non-adiabatic environments.

In such cases, higher-order field terms can no longer be neglected, and Einstein-domain closure breaks down.

6.3.7 Relation to the GR Limit

This condition does not deny that curvature exists in the Einstein domain. On the contrary, curvature is precisely the effective macroscopic manifestation of the coarse-grained Φ structure. What is excluded is not curvature itself, but uncontrolled microscopic or mesoscopic field structure that would prevent a clean Einstein-form closure. Thus, the smooth Φ-field condition should be understood as a requirement of controlled curvature emergence, not of vanishing field gradients.

6.3.8 Summary Statement

The smooth Φ-field condition requires that the coarse-grained substrate vary slowly and remain free of strong higher-gradient structure across the averaging domain, so that the effective metric dynamics can be truncated to leading Einstein form and all residual field-structure effects remain subleading.

6.4 Adiabatic Evolution Condition

Scope: Require that the temporal evolution of the coarse-grained Φ substrate and its coupling response be sufficiently slow that inertial time-dependent corrections do not enter the effective gravitational dynamics at leading order.

6.4.1 Statement of Condition

Let $\Phi(\mathbf{x}, t)$ denote the coarse-grained scalar substrate field and let τ denote the characteristic temporal coarse-graining scale associated with the effective continuum description.

The adiabatic evolution condition requires that the local temporal variation of the substrate be slow compared with the response timescale of the effective gravitational system. In normalized form, this requires:

$$\frac{\sqrt{h^{\mu\nu}\,\nabla_\mu\Phi\,\nabla_\nu\Phi}}{|\,\Phi\,|}\,L \ll 1$$

$$\frac{\sqrt{h^{\mu\nu}h^{\alpha\beta}\,\nabla_\mu\nabla_\alpha\Phi\,\nabla_\nu\nabla_\beta\Phi}}{|\,\Phi\,|}\,L^2 \ll 1$$

Where higher-order temporal response is relevant, Einstein closure further requires suppression of rapid temporal curvature in the field evolution, schematically:

$$|\frac{\partial_t^2 \Phi}{\Phi}| \tau^2 \ll 1$$

If the effective coupling $S(\Sigma)$ is itself time-dependent through evolving density, the same adiabatic requirement must hold for the coupling sector:

$$|\frac{\partial_t S}{S}| \tau \ll 1$$

Together, these conditions define the temporal slow-variation regime required for Einstein-domain closure.

6.4.2 Physical Meaning

This condition states that the effective gravitational environment must evolve slowly enough that the substrate remains in near-quasi-static local adjustment over the averaging timescale.

It does **not** require the universe, the matter distribution, or the Φ field to be exactly static. Rather, it requires that:

- temporal evolution is gradual rather than abrupt,
- local substrate response remains close to equilibrium across the coarse-graining interval,
- rapidly propagating or transient dynamical effects do not dominate the leading-order closure.

Physically, the system may evolve, but it must do so slowly enough that the emergent continuum description tracks that evolution without substantial lag, overshoot, or explicit dynamical memory effects.

6.4.3 Role in Einstein Closure

This condition is necessary because rapid time dependence generically introduces additional dynamical terms that are not present in the Einstein-domain limit.

If the substrate evolves too quickly, then:

- explicit time-derivative terms survive coarse-graining at leading order,
- local source response may cease to be quasi-static,
- retardation, relaxation, or non-local temporal effects may become important,
- the curvature–source relation acquires non-Einstein dynamical corrections.

By imposing adiabatic evolution, these time-dependent corrections are suppressed and may be consistently absorbed into the remainder tensor $\Delta_{\mu\nu}$, leaving Einstein-form dynamics as the leading-order closure.

6.4.4 Distinction Between Evolution and Breakdown

The adiabatic condition must not be confused with a requirement of frozen dynamics. General Relativity itself describes evolving gravitational systems. The question here is not whether evolution occurs, but whether the underlying Φ substrate evolves in a manner sufficiently slow that its detailed time dependence need not appear explicitly in the leading-order effective equations.

Thus:

- slow evolution is compatible with Einstein-domain dynamics,
- rapid substrate evolution is not.

The Einstein domain is therefore an adiabatic dynamical regime, not a static one. This adiabatic dynamical regime is one where the system evolves slowly enough that, at each moment, it remains very close to a local equilibrium or steady-state configuration.

6.4.5 Relationship to Coarse-Graining

As in Section 6.3, this condition is defined relative to the temporal averaging scale τ.

This is important. The requirement is not that all microscopic time dependence vanish, but that any fast substrate-level fluctuations average out over the interval used to define the effective continuum theory.

Accordingly, short-timescale microscopic oscillations or adjustment processes may exist, provided they do not survive coarse-graining as leading-order contributions to the macroscopic field equations.

6.4.6 Failure Mode

The condition fails when one or more of the quantities

$$|\frac{\partial_t \Phi}{\Phi}|\,\tau, \qquad |\frac{\partial_t^2 \Phi}{\Phi}|\,\tau^2, \qquad |\frac{\partial_t S}{S}|\,\tau$$

cease to be small.

This may occur in regimes involving:

- rapid threshold approach or threshold crossing,
- strongly time-dependent matter redistribution,
- violent or transient substrate response,
- environments in which the Φ field cannot maintain near-quasi-static adjustment.

In such cases, time-dependent correction terms become dynamically important, and Einstein-domain closure no longer holds at leading order.

6.4.7 Relation to the Correction Tensor

Within the Einstein domain, residual time-dependent effects are subleading and may be collected into the correction tensor $\Delta_{\mu\nu}$. Outside that domain, these terms can no longer be treated as perturbative. They become part of the dominant effective dynamics, signalling departure from GR correspondence and the onset of genuinely non-Einstein behaviour. Thus, adiabaticity is one of the key assumptions that governs whether $\Delta_{\mu\nu}$ is negligible or dynamically active.

6.4.8 Summary Statement

The adiabatic evolution condition requires that the coarse-grained Φ substrate and its coupling response evolve slowly relative to the effective gravitational response timescale, so that explicit time-dependent substrate corrections remain subleading and Einstein-form dynamics can emerge at leading order.

6.5 Universality Condition

Scope: Require that, within the Einstein domain, matter responds to the coarse-grained Φ-mediated gravitational sector in an effectively species-independent manner, so that a single geometric description and a single effective source coupling are valid to leading order.

6.5.1 Statement of Condition

Let the matter sector consist of the coarse-grained fields and sources contributing to the effective stress-energy tensor $T_{\mu\nu}^{\mathrm{eff}}$. The universality condition requires that, throughout the Einstein domain, the response of matter to the emergent gravitational sector be effectively identical to leading order across all relevant matter species and configurations.Operationally, this means that any species-dependence, composition-dependence, or environment-dependent variation in the coupling of matter to the effective metric must be perturbatively suppressed, so that the leading-order dynamics may be written using a single effective gravitational coupling $G_{\mathrm{eff}}$and a single geometric structure $g_{\mu\nu}^{\mathrm{eff}}$.

Equivalently, Einstein closure requires that non-universal response terms satisfy a suppression condition of the form

$$\epsilon_{\text{non-univ}} \ll 1$$

where $\epsilon_{\text{non-univ}}$ denotes the dimensionless magnitude of composition- or species-dependent deviations relative to the leading universal coupling behaviour. These conditions are expressed as scalar invariants and are therefore independent of coordinate choice, ensuring that the definition of the Einstein domain is fully covariant.

$$\epsilon = \max\left(\frac{\sqrt{h^{\mu\nu}\nabla_\mu S\nabla_\nu S}}{|\,S\,|}L,\ \frac{|\,u^\mu\nabla_\mu S\,|}{|\,S\,|}\tau,\ \frac{\sqrt{h^{\mu\nu}\nabla_\mu \Phi\nabla_\nu \Phi}}{|\,\Phi\,|}L,\ \frac{|\,u^\mu\nabla_\mu \Phi\,|}{|\,\Phi\,|}\tau,\ \epsilon_{\text{non-univ}}\right)$$

$$\epsilon \ll 1$$

This dimensionless parameter ϵ provides a fully covariant measure of deviation from ideal closure, with the Einstein domain defined by the condition $\epsilon \ll 1$.

6.5.2 Physical Meaning

This condition does not assert that universality is fundamental in GETT. Rather, it asserts that universality emerges as an excellent approximation within the Einstein domain. In GETT, matter couples through a substrate-based mechanism involving the Φ field and the density-regulated interaction structure. In principle, departures from perfect universality may arise outside the GR regime. However, for Einstein-domain closure to hold, such departures must be negligible across the coarse-graining domain.

Physically, this means that:

- all relevant matter species experience the same effective geometric background to leading order,
- local free-fall behaviour is effectively common,
- no composition-dependent gravitational response survives into the leading continuum description.

Thus, the Einstein domain is one in which effective universality is recovered, even if universality is not taken to be ontologically exact in all regimes.

6.5.3 Role in Einstein Closure

This condition is essential because Einstein-form dynamics require a single geometric sector coupled to a single effective source. If different matter species responded differently at leading order, then:

- no single effective metric could represent the motion of all matter consistently,
- the source sector would fail to collapse into a single universal $T_{\mu\nu}^{\text{eff}}$,
- the curvature–source relation would acquire composition-dependent terms,
- the Einstein-form closure would fail.

Accordingly, effective universality is required not as a philosophical preference, but as a structural necessity for a GR-like description.

6.5.4 Relationship to the Mid-Density Regime

The universality condition is naturally linked to the mid-density and weak-variation conditions established in Sections 6.1 and 6.2. When $S(\Sigma)$ is approximately constant and its gradients are negligible across the averaging domain, matter experiences an effectively common coupling environment. Under these conditions, any composition- or species-dependent response differences are suppressed, and the leading-order gravitational interaction becomes effectively universal. Thus, in GETT, universality is not assumed independently. It arises as a consequence of the same regime conditions that permit Einstein closure more generally.

6.5.5 Domain Qualification

The universality condition is satisfied when:

- the system lies within the mid-density plateau,
- coupling variation is negligible over the coarse-graining domain,
- no threshold-induced modulation differentiates the response of matter sectors at leading order,
- all relevant species probe the same effective continuum geometry to observational tolerance.

Within this regime, matter may be treated as sourcing and responding to a common Einstein-domain metric.

6.5.6 Failure Mode

The condition fails when composition-dependent or environment-sensitive response terms become significant at leading order.

This may occur when:

- the system approaches or crosses a density threshold,
- coupling modulation becomes active,
- non-universal substrate response is no longer perturbatively suppressed,
- different matter sectors probe measurably different effective couplings or geometric response.

In such cases:

- effective free-fall universality is lost,
- the single-metric approximation breaks down,
- non-Einstein corrections can no longer be absorbed as negligible contributions.

Accordingly, Einstein-domain closure fails.

6.5.7 Relation to the Equivalence Principle

Within GETT, the equivalence-principle-like behaviour recovered in the Einstein domain is an emergent regime property, not a universally fundamental axiom. This distinction is crucial. The present condition requires only that effective universality hold to leading order within the domain relevant to GR correspondence. It does not require that exact universality persist in low-density, threshold, or ultra-high-density regimes where GETT predicts domain breakdown or modified behaviour. Thus, the universality condition is entirely consistent with the broader GETT position that equivalence-principle behaviour is domain-limited rather than absolute.

6.5.8 Summary Statement

The universality condition requires that, within the Einstein domain, all relevant matter sectors respond to the emergent gravitational background in an effectively species-independent manner, so that a single effective metric, a single effective source tensor, and a single leading-order gravitational coupling are sufficient to recover Einstein-form dynamics.

6.6 Closure Statement

Scope: State the formal condition under which the coarse-grained Φ-substrate dynamics admit Einstein-form dynamical closure and define the structure and suppression of non-Einstein corrections.

6.6.1 Hypothesis Set

Let the coarse-grained GETT system satisfy the conditions defined in Sections 6.1–6.5 over a spacetime domain characterised by averaging scales L and τ. Specifically:

(H1) Mid-density regime: $S(\Sigma) \approx S_0$, with fractional deviations $\ll 1$	**(H4) Adiabatic evolution:** $\mid \frac{\partial_t \Phi}{\Phi} \mid \tau \ll 1 \qquad \mid \frac{\partial_t^2 \Phi}{\Phi} \mid \tau^2 \ll 1$
(H2) Weak coupling variation: $\mid \frac{\nabla S}{S} \mid L \ll 1 \qquad \mid \frac{\partial_t S}{S} \mid \tau \ll 1$	**(H5) Effective universality:** $\epsilon_{\text{non-univ}} \ll 1$
(H3) Smooth Φ field: $\mid \frac{\nabla \Phi}{\Phi} \mid L \ll 1 \qquad \mid \frac{\partial_t \Phi}{\Phi} \mid \tau \ll 1$ $\mid \frac{\nabla\nabla \Phi}{\Phi} \mid L^2 \ll 1$	the full covariant ε was first assembled in Section 6.5.1.

These conditions collectively define the **Einstein-domain regime of GETT.**

6.6.2 Closure Statement

Under hypotheses (H1)–(H5), the coarse-grained dynamics of the coupled matter–Φ system admit a truncation in which:

1. The effective stress-energy tensor $T_{\mu\nu}^{\text{eff}}$ is well-defined and locally conserved,

$$\nabla_\mu T_{\mu\nu}^{\text{eff}} = 0$$

2. The effective metric $g_{\mu\nu}^{\text{eff}}$supports a standard curvature construction,
3. The curvature–source relation takes the form

$$G_{\mu\nu} = 8\pi G_{\text{eff}} T_{\mu\nu}^{\text{eff}} + \Delta_{\mu\nu}$$

   where G_{eff} is approximately constant over the domain, and $\Delta_{\mu\nu}$ collects all non-Einstein contributions [1,2,5].

6.6.3 Suppression of Non-Einstein Terms

Under the same hypotheses, the correction tensor satisfies

$$\Delta_{\mu\nu} = O(\epsilon), \qquad \epsilon \ll 1$$

where ϵ is a dimensionless control parameter representing the combined magnitude of:

- coupling variation,
- Φ-field gradients and higher derivatives,
- non-adiabatic temporal effects,
- non-universal matter response.

Accordingly, to leading order in ϵ,

$$G_{\mu\nu} = 8\pi G_{\text{eff}} T_{\mu\nu}^{\text{eff}} + O(\epsilon)$$

In the strict Einstein-domain limit $\epsilon \to 0$, the correction tensor vanishes:

$$\Delta_{\mu\nu} \to 0$$

and the Einstein field equations are recovered exactly in form.

6.6.4 Emergence of the Effective Gravitational Constant

Within this regime, the effective gravitational coupling G_{eff} arises from the underlying Φ–matter interaction structure and satisfies:

- approximate constancy across the domain,
- stability under small perturbations,
- independence from local composition at leading order.

Thus, G_{eff} functions as the emergent coupling parameter governing Einstein-domain dynamics.

6.6.5 Interpretation as a Leading-Order Closure

The Einstein-form equation obtained above is not fundamental in GETT. It is the leading-order closure of the full Φ-substrate dynamics under the stated hypotheses. All deviations from GR are encoded in $\Delta_{\mu\nu}$, which becomes dynamically relevant only when one or more of the closure conditions (H1)–(H5) are violated.

Accordingly:

- GR corresponds to the regime $\epsilon \ll 1$,
- departures from GR correspond to regimes where $\epsilon$is not small.

6.6.6 Domain-Limited Validity

The closure result holds only within the Einstein-domain regime defined by the hypotheses above.

Outside this regime:

- coupling modulation may become significant,
- higher-gradient Φ structure may dominate,
- non-adiabatic dynamics may emerge,
- universality may break down,

and the Einstein-form truncation is no longer valid. Thus, Einstein-domain dynamics are conditionally valid, not universally fundamental.

6.6.7 Summary Statement

Under smooth, weakly varying, adiabatic, and effectively universal conditions in the mid-density regime, the coarse-grained Φ-substrate dynamics admit a leading-order truncation in which spacetime curvature evolves according to Einstein-form field equations with a stable effective gravitational coupling, while all non-Einstein contributions are perturbatively suppressed.

6.7. Section 6 Summary Grid

Table 6, next page, shows the complete set of covariant closure conditions required for Einstein-domain dynamics, together with their physical roles and the resulting constraints on the effective theory. Collectively, these conditions define a single dimensionless control parameter $\epsilon \ll 1$, under which all non-Einstein contributions are perturbatively suppressed, and the dynamical system closes in Einstein form.

Component	Requirement (Covariant Form)	Physical Role	Outcome for Dynamics	Status
Mid-density coupling	$S(\Sigma) \approx S_0$, with $dS/d\Sigma \approx 0$	Eliminates coupling variation	Prevents additional source terms	PASS
Weak spatial coupling gradients	$\frac{\sqrt{h^{\mu\nu}\nabla_\mu S \nabla_\nu S}}{\mid S \mid} L \ll 1$	Constrains spatial variation of the coupling across the coarse-graining scale.	Prevents gradient-driven coupling corrections from entering the leading-order equations.	PASS
Weak temporal coupling variation	$\frac{\mid u^\mu \nabla_\mu S \mid}{\mid S \mid} \tau \ll 1$	Constrains temporal variation of the coupling across the averaging timescale.	Suppresses time-dependent coupling corrections.	PASS
Smooth Φ-field (first derivative)	$\frac{\sqrt{h^{\mu\nu}\nabla_\mu \Phi \nabla_\nu \Phi}}{\mid \Phi \mid} L \ll 1$	Requires the coarse-grained substrate to vary slowly in space.	Ensures a smooth continuum field suitable for emergent metric description.	PASS
Smooth Φ-field (second derivative)	$\frac{\sqrt{h^{\mu\nu} h^{\alpha\beta}\, \nabla_\mu \nabla_\alpha \Phi\, \nabla_\nu \nabla_\beta \Phi}}{\mid \Phi \mid} L^2 \ll 1$	Suppresses sharp local structure and higher-gradient leakage.	Keeps higher-derivative substrate terms subleading.	PASS
Adiabatic evolution	$\frac{\sqrt{h^{\mu\nu} h^{\alpha\beta}\, \nabla_\mu \nabla_\alpha \Phi\, \nabla_\nu \nabla_\beta \Phi}}{\mid \Phi \mid} L^2 \ll 1$	Suppresses sharp local structure and higher-gradient leakage.	Keeps higher-derivative substrate terms subleading.	PASS
Effective Universality	$\frac{\mid u^\mu \nabla_\mu \Phi \mid}{\mid \Phi \mid} \tau \ll 1$	Ensures identical matter response. Requires slow temporal evolution of the substrate.	Guarantees single effective source. Eliminates leading-order non-adiabatic corrections	PASS
Coarse-graining validity	$\epsilon_{\text{non-univ}} \ll 1$	Requires effectively species-independent matter response. Controls averaging residuals	Allows a single effective metric and a single effective source tensor. Ensures commutator terms are suppressed	PASS
Potential regularity	$V'(\Phi_0) \approx 0, V''(\Phi_0) = m_\Phi^2 > 0$	Requires local stability of the scalar potential in the Einstein domain. Ensures stable local configuration	Supports well-defined fluctuations and a stable substrate background.	PASS
Closure parameter	$\epsilon \ll 1$	Collects all closure constraints into a single control parameter.Unified control measure	Defines the Einstein domain as a controlled perturbative regime.	PASS
Closure result	All conditions above satisfied simultaneously.	Defines the theorem hypothesis set for Einstein closure.	$\Delta_{\mu\nu} = O(\epsilon)$	PASS
Einstein-domain outcome	$\epsilon \to 0$ or $\epsilon \ll 1$ at leading order	Leading-order dynamics – defines the regime in which the effective system becomes self-contained.	$G_{\mu\nu} = 8\pi G_{\text{eff}}\, T_{\mu\nu}^{\text{eff}}$	PASS

Table 6. Einstein-Domain Closure Conditions and Outcomes.

7. Derivation of Einstein-Domain Dynamics

This section demonstrates that, under the closure conditions established in Section 6, the effective metric arising from the coarse-grained Φ–matter system obeys an Einstein-form dynamical relation. The goal is not to assume General Relativity, but to show that the macroscopic variables constructed in Sections 4 and 5 admit a leading-order curvature–source closure of the form

$$G_{\mu\nu} = 8\pi G_{\text{eff}}\, T_{\mu\nu}^{\text{eff}} + \Delta_{\mu\nu}$$

where:

- $G_{\mu\nu}$ is the Einstein tensor of the effective metric,
- $T_{\mu\nu}^{\text{eff}}$ is the conserved effective stress-energy tensor,
- G_{eff} is an emergent gravitational coupling,
- $\Delta_{\mu\nu}$ collects all non-Einstein corrections.

The derivation proceeds in layers:

1. relate the effective metric to the coarse-grained substrate,
2. construct the associated curvature tensors,
3. insert the effective source structure,
4. isolate the residual non-Einstein terms,
5. show that these residuals are suppressed in the Einstein domain.

This is the point at which GETT must show not merely that geometry emerges, but that geometry evolves in Einstein form [5,6] within its domain of validity.

7.1 Metric–Substrate Link

Scope: Establish the dynamical relationship between the effective metric and the coarse-grained Φ-substrate, so that curvature may be interpreted as the macroscopic geometric encoding of underlying field behaviour.

7.1.1 Starting Point

Paper 4.1 established that, in the geometrisation regime, the macroscopic behaviour of the Φ substrate can be represented by an effective metric $g_{\mu\nu}^{\text{eff}}$. That result provided the kinematical basis for emergent spacetime structure.

The present task is dynamical: to connect the evolution of this effective metric to the underlying coarse-grained Φ–matter system. Accordingly, the effective metric is not introduced as an independent fundamental entity. It is understood as a macroscopic encoding of the state of the substrate, and therefore as a functional of the coarse-grained variables:

$$g_{\mu\nu}^{\text{eff}} = g_{\mu\nu}^{\text{eff}}(\Phi_{\text{eff}}, \nabla_\alpha \Phi_{\text{eff}}, S_{\text{eff}}, T_{\alpha\beta}^{\text{eff}}, \dots)$$

This expression is schematic, but its meaning is precise: the effective metric is determined by the averaged substrate configuration and its matter-coupled response.

7.1.2 Physical Interpretation

Within GETT, the metric is not the primary ontology. The primary ontology is the real Φ substrate and its interaction with matter.

The effective metric arises because, after coarse-graining:

- the detailed microscopic structure is not directly resolved,
- matter responds to the averaged substrate as if moving in a curved geometry,
- the causal, interval, and trajectory structure can be encoded in geometric form.

Thus, the effective metric represents the macroscopic measurement structure induced by the substrate, not a fundamental spacetime substance.

7.1.3 Dynamical Dependence

Since the substrate evolves according to the field equation of Section 3.1 and the coupling structure of Section 3.3, the effective metric necessarily inherits this dynamics indirectly.

That is, changes in:

- the coarse-grained field amplitude Φ_{eff},
- its gradient structure,
- the local coupling state S_{eff},
- and the effective source content $T_{\mu\nu}^{\text{eff}}$,

produce corresponding changes in $g_{\mu\nu}^{\text{eff}}$.

Schematically,

$$\delta g_{\mu\nu}^{\text{eff}} = \frac{\partial g_{\mu\nu}^{\text{eff}}}{\partial \Phi_{\text{eff}}}\,\delta\Phi_{\text{eff}} + \frac{\partial g_{\mu\nu}^{\text{eff}}}{\partial(\nabla_\alpha \Phi_{\text{eff}})}\,\delta(\nabla_\alpha \Phi_{\text{eff}}) + \frac{\partial g_{\mu\nu}^{\text{eff}}}{\partial S_{\text{eff}}}\,\delta S_{\text{eff}} + \cdots$$

This establishes the essential point: metric dynamics are induced dynamics, inherited from the evolving coarse-grained substrate.

7.1.4 Einstein-Domain Simplification

Under the closure conditions of Section 6, the dependence of the effective metric simplifies substantially.

In particular:

- $S_{\text{eff}} \approx S_0$, so coupling dependence becomes approximately constant,
- Φ_{eff} varies smoothly, so higher-gradient structure is suppressed,
- evolution is adiabatic, so rapid time-dependent substrate corrections are absent,
- non-universal response terms are negligible.

Accordingly, in the Einstein domain the metric dependence reduces to its leading smooth, universal form, and all higher-order substrate-sensitive dependence may be collected into subleading corrections. This makes it possible to treat the effective metric as a regular continuum geometric field whose dynamics admit closure.

7.1.5 Consequence for Curvature Construction

Once the metric is recognised as a smooth functional of the coarse-grained substrate, all standard curvature objects may be built from it in the usual way.

That is, from $g_{\mu\nu}^{\text{eff}}$ one defines:

- the Levi-Civita connection,
- the Riemann curvature tensor,
- the Ricci tensor,
- the Ricci scalar,
- the Einstein tensor.

The geometrical side of the Einstein-form equation is therefore not postulated independently: it is the macroscopic curvature structure associated with the substrate-induced effective metric.

7.1.6 Conceptual Result

This subsection establishes the first step of the dynamical correspondence:

the effective metric is a coarse-grained dynamical encoding of the Φ–matter substrate, so its evolution is determined by the evolution of the underlying field system.

This is what makes an Einstein-domain closure possible in principle.

7.1.7 Summary Statement

In the Einstein domain, the effective metric is a smooth, coarse-grained functional of the Φ-substrate and its matter-coupled state, so that the evolution of geometry is inherited from the underlying substrate dynamics and may be represented by standard curvature tensors at the macroscopic level.

7.2 Curvature Construction

Scope: Construct the full set of curvature tensors from the effective metric $g_{\mu\nu}^{\text{eff}}$, establishing the geometric side of the dynamical equations as a well-defined macroscopic structure. The resulting curvature–source relation is consistent with the standard variational formulation of General Relativity, in which the Einstein field equations arise from the Einstein–Hilbert action [13].

7.2.1 Levi–Civita Connection

Given the effective metric $g_{\mu\nu}^{\text{eff}}$, we define the unique torsion-free, metric-compatible connection (Levi–Civita connection):

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} g^{\rho\sigma}\left(\partial_{\mu} g_{\nu\sigma} + \partial_{\nu} g_{\mu\sigma} - \partial_{\sigma} g_{\mu\nu}\right)$$

where all quantities are constructed from $g_{\mu\nu}^{\text{eff}}$.

This connection defines:

- parallel transport,
- covariant differentiation,
- and the local geometric structure of spacetime.

7.2.2 Riemann Curvature Tensor

From the connection, the Riemann curvature tensor is defined as:

$$R^{\rho}{}_{\sigma\mu\nu} = \partial_{\mu}\Gamma^{\rho}_{\nu\sigma} - \partial_{\nu}\Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$$

This tensor encodes:

- the failure of parallel transport to be path-independent,
- the intrinsic curvature of the effective spacetime.

7.2.3 Ricci Tensor and Scalar

Contracting the Riemann tensor yields the Ricci tensor:

$$R_{\mu\nu} = R^{\rho}{}_{\mu\rho\nu}$$

and further contraction gives the Ricci scalar:

$$R = g^{\mu\nu}R_{\mu\nu}$$

These quantities summarise the curvature relevant for dynamical gravitational behaviour at the macroscopic level.

7.2.4 Einstein Tensor

The Einstein tensor is defined as:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$$

This tensor has the key property:

$$\nabla^{\mu}G_{\mu\nu} = 0$$

which follows from the contracted Bianchi identity.

7.2.5 Role in Dynamical Closure

The construction above provides the complete geometric structure required for Einstein-form dynamics.

Crucially:

- these tensors are not introduced independently of the Φ-substrate,
- they are constructed from the effective metric derived from the coarse-grained system,
- their properties (e.g. covariant conservation of $G_{\mu\nu}$) are inherited from the geometric structure of that metric.

Thus, the left-hand side of the Einstein-form equation is fully defined once the effective metric is specified.

7.2.6 Compatibility with Effective Source

From Section 5, the effective stress-energy tensor satisfies:

$$\nabla^{\mu} T_{\mu\nu}^{\text{eff}} \approx 0$$

From geometry:

$$\nabla^{\mu} G_{\mu\nu} = 0$$

This structural compatibility is essential:

- it ensures that a curvature–source relation can be consistently defined,
- it guarantees that any dynamical equation relating $G_{\mu\nu}$ and $T_{\mu\nu}^{\text{eff}}$ is mathematically consistent,
- and it prepares the ground for Einstein-form closure.

This guarantees that the emergent Einstein-domain dynamics satisfy the Bianchi identity $\nabla^{\mu} G_{\mu\nu} = 0$, ensuring consistency with covariant stress–energy conservation.

7.2.7 Einstein-Domain Interpretation

Within GETT, these curvature tensors are interpreted as:

- the geometric encoding of coarse-grained Φ-substrate behaviour,
- not as fundamental entities independent of physical substrate.

Thus:

- curvature reflects averaged tension structure,
- the Einstein tensor reflects the macroscopic response of the substrate,
- and geometry becomes a representation of underlying physics.

7.2.8 Summary Statement

The geometric structure introduced here follows the standard Levi-Civita connection and curvature tensor framework of General Relativity, ensuring full compatibility with established formulations. The effective metric derived from the coarse-grained Φ–matter system admits a complete geometric construction – including connection, curvature tensors, and Einstein tensor – providing the fully defined geometric side of the dynamical equations required for Einstein-domain closure.

7.3 Source Coupling

Scope: Establish the leading-order coupling between the effective curvature tensors and the coarse-grained effective stress-energy tensor, defining the source structure of Einstein-domain dynamics.

7.3.1 Need for a Curvature–Source Relation

Sections 7.1 and 7.2 established that the coarse-grained Φ–matter system admits an effective metric $g_{\mu\nu}^{\text{eff}}$ and the associated curvature tensors $R^{\rho}{}_{\sigma\mu\nu}$, $R_{\mu\nu}$, R, and $G_{\mu\nu}$.

Section 5 established that the same coarse-grained system admits a single effective stress-energy tensor $T_{\mu\nu}^{\text{eff}}$, which is conserved to leading order in the Einstein domain:

$$\nabla^{\mu} T_{\mu\nu}^{\text{eff}} \approx 0$$

The remaining task is therefore to identify the dynamical relation linking these two structures.

For Einstein-domain correspondence to hold, curvature must couple to the effective source in a local tensorial form whose leading behaviour matches the Einstein field equations [1,2,5,7].

7.3.2 Leading-Order Coupling Ansatz

Since both $G_{\mu\nu}$ and $T_{\mu\nu}^{\text{eff}}$ are symmetric rank-2 tensors defined on the same effective spacetime, and both satisfy covariant conservation to leading order, the most general leading-order local closure consistent with the Section 2 requirements is of the form:

$$G_{\mu\nu} = \kappa_{\text{eff}} T_{\mu\nu}^{\text{eff}} + \Delta_{\mu\nu}$$

where:

- κ_{eff} is the effective curvature–source coupling coefficient,
- $\Delta_{\mu\nu}$ collects all terms not absorbed into the leading Einstein-form closure.

It is convenient to write

$$\kappa_{\text{eff}} = 8\pi G_{\text{eff}}$$

so that

$$G_{\mu\nu} = 8\pi G_{\text{eff}} T_{\mu\nu}^{\text{eff}} + \Delta_{\mu\nu}$$

This is the candidate Einstein-domain dynamical equation.

7.3.3 Meaning of G_{eff}

The quantity G_{eff} is not inserted arbitrarily. It represents the effective macroscopic coupling strength that emerges from the underlying Φ–matter interaction structure after coarse-graining.

Its physical origin lies in:

- the baseline interaction strength λ_0,
- the local modulation state $S(\Sigma)$,
- the response properties of the Φ substrate,
- and the averaging procedure that produces the continuum limit.

In the Einstein domain, where $S(\Sigma) \approx S_0$ and its variation is weak, G_{eff} becomes approximately constant over the averaging region and plays the role of the gravitational coupling appearing in the macroscopic field equations.

7.3.4 Why the Leading Coupling Must Be Local

The curvature–source relation must be local at leading order because:

- the effective geometry is defined pointwise from coarse-grained fields,
- the effective stress-energy tensor is defined as a local continuum source,
- and GR correspondence requires a local tensorial closure rather than an explicitly non-local constitutive law.

Any residual non-locality introduced by unresolved substrate structure, averaging artifacts, or delayed response cannot appear in the leading Einstein-domain term. Such contributions must instead be relegated to $\Delta_{\mu\nu}$. This ensures that the leading-order equation retains the local structure characteristic of Einstein dynamics.

7.3.5 Why the Coupling Is Tensorially Consistent

This form is structurally consistent for three reasons:

1. **Tensor Rank and Symmetry**
   Both $G_{\mu\nu}$ and $T_{\mu\nu}^{\text{eff}}$ are symmetric rank-2 tensors.
2. **Conservation Compatibility**
   In the Einstein domain,

   $$\nabla^{\mu} G_{\mu\nu} = 0, \nabla^{\mu} T_{\mu\nu}^{\text{eff}} \approx 0$$

   so the leading coupling is dynamically consistent.
3. **Continuum Closure**
   The coarse-grained system admits no lower-order local tensorial object with the same physical role that could replace $T_{\mu\nu}^{\text{eff}}$ as the macroscopic source.

Thus, the Einstein-form coupling is not merely convenient; it is the natural leading-order closure compatible with the continuum geometric description.

7.3.6 Domain-Limited Nature of the Coupling

The relation

$$G_{\mu\nu} = 8\pi G_{\text{eff}}\, T_{\mu\nu}^{\text{eff}} + \Delta_{\mu\nu}$$

must be understood as domain-limited.

It holds only when the conditions of Section 6 are satisfied, namely when:

- coupling variation is weak,
- the Φ field is smooth,
- evolution is adiabatic,

- and matter response is effectively universal.

Outside that domain:

- G_{eff} may no longer be approximately constant,
- local tensorial closure may cease to hold at leading order,
- and $\Delta_{\mu\nu}$ may become dynamically significant.

Thus, this equation is an Einstein-domain closure law, not a universal fundamental identity.

7.3.7 Physical Interpretation

This relation states that, in the Einstein domain:

- the effective curvature of spacetime is sourced by the total coarse-grained energy–momentum content of the matter–Φ system,
- with coupling strength determined by the locally stable substrate response.

In **GR language**, curvature is sourced by stress-energy.

In **GETT language**, that same relation is reinterpreted as:

- curvature is the macroscopic geometric encoding of substrate tension dynamics,
- and the source term is the coarse-grained energy–momentum content of the coupled substrate–matter system.

So the form is Einsteinian, but the ontology is GETT.

7.3.8 Summary Statement

The effective curvature tensors constructed from the emergent metric couple at leading order to the conserved coarse-grained stress-energy tensor through an approximately constant emergent gravitational coupling G_{eff}, yielding the Einstein-form source relation up to a residual correction tensor $\Delta_{\mu\nu}$.

7.4 Residual Decomposition

Scope: Isolate and classify the non-Einstein contributions contained in $\Delta_{\mu\nu}$, showing that all deviations from Einstein-form dynamics arise from identifiable physical effects in the underlying Φ–matter system.

7.4.1 Definition of the Residual Tensor

From Section 7.3, the Einstein-domain dynamical relation was written in the form

$$G_{\mu\nu} = 8\pi G_{\text{eff}}\, T_{\mu\nu}^{\text{eff}} + \Delta_{\mu\nu}$$

The tensor $\Delta_{\mu\nu}$ is defined as the remainder after extracting the leading local Einstein-form coupling between curvature and effective source.

Equivalently,

$$\Delta_{\mu\nu} \equiv G_{\mu\nu} - 8\pi G_{\text{eff}}\, T_{\mu\nu}^{\text{eff}}$$

This tensor therefore contains all corrections to exact Einstein closure.

7.4.2 Origin of Residual Terms

The correction tensor $\Delta_{\mu\nu}$arises because the coarse-grained Φ–matter system is not, in general, identical to an ideal Einsteinian continuum.

Its contributions originate from four principal sources:

1. **Coupling-modulation effects**
   arising from spatial or temporal variation in $S(\Sigma)$,
2. **Higher-gradient Φ-field structure**
   arising from unresolved first- and second-derivative field terms beyond leading smooth behaviour,
3. **Non-adiabatic temporal response**
   arising from explicit time-dependent substrate dynamics that do not average out,
4. **Residual non-universal response**
   arising from composition- or sector-dependent coupling effects not fully suppressed in the continuum limit.

Thus, $\Delta_{\mu\nu}$ is not an arbitrary mathematical remainder; it is the macroscopic representation of physically identifiable deviations from Einstein-domain conditions.

7.4.3 Schematic Decomposition

It is therefore useful to write $\Delta_{\mu\nu}$schematically as

$$\Delta_{\mu\nu} = \Delta_{\mu\nu}^{(S)} + \Delta_{\mu\nu}^{(\Phi\text{-grad})} + \Delta_{\mu\nu}^{(\text{adia})} + \Delta_{\mu\nu}^{(\text{non-univ})} + \Delta_{\mu\nu}^{(\text{avg})},$$

where:

- $\Delta_{\mu\nu}^{(S)}$: corrections from coupling variation,
- $\Delta_{\mu\nu}^{(\Phi\text{-grad})}$: corrections from higher-order Φ gradients,
- $\Delta_{\mu\nu}^{(\text{adia})}$: corrections from non-adiabatic evolution,
- $\Delta_{\mu\nu}^{(\text{non-univ})}$: corrections from non-universal response,
- $\Delta_{\mu\nu}^{(\text{avg})}$: corrections from averaging residuals, including non-commutativity of averaging and differentiation.

This decomposition is schematic rather than yet fully expanded, but it identifies the complete structure of the residual sector.

7.4.4 Coupling-Modulation Residuals

The first class of corrections arises when the coupling modulation function is not locally constant.

When

$$|\frac{\nabla S}{S}|L \not\ll 1 \qquad \text{or} \qquad |\frac{\partial_t S}{S}|\tau \not\ll 1$$

terms generated by coupling variation survive coarse-graining at leading or next-to-leading order.

These terms alter the effective curvature–source relation by introducing:

- position-dependent coupling structure,
- explicit derivative contributions,
- departures from a single locally constant G_{eff}.

Such effects are collected into $\Delta_{\mu\nu}^{(S)}$.

7.4.5 Higher-Gradient Φ Residuals

The second class arises from unresolved structure in the Φ substrate itself. When the field contains significant higher-order spatial or temporal variation across the averaging domain, corrections arise from terms schematically involving

$$\nabla_\mu \Phi\, \nabla_\nu \Phi, \nabla_\mu \nabla_\nu \Phi, \partial_t^2 \Phi$$

beyond the smooth leading-order continuum truncation.

These terms reflect:

- local substrate structure,
- anisotropic tension effects,
- curvature contributions not fully reducible to perfect-fluid form.

They are collected into $\Delta_{\mu\nu}^{(\Phi\text{-grad})}$.

7.4.6 Non-Adiabatic Residuals

The third class arises when the Φ substrate or coupling sector evolves too rapidly for quasi-static closure.

If

$$|\frac{\partial_t \Phi}{\Phi}|\tau \not\ll 1 \qquad \text{or} \qquad |\frac{\partial_t^2 \Phi}{\Phi}|\tau^2 \not\ll 1$$

then explicit time-dependent response terms remain in the effective equations.

These may represent:

- relaxation effects,
- delayed substrate response,
- transient dynamical behaviour,
- non-equilibrium evolution.

Such contributions are collected into $\Delta_{\mu\nu}^{(\text{adia})}$.

7.4.7 Non-Universal Residuals

The fourth class arises if matter does not respond identically to the emergent gravitational sector at leading order. When effective universality fails, the continuum source cannot be represented perfectly by a single universal coupling to a single metric. Residual composition- or sector-dependent effects then remain, producing corrections collected into

$$\Delta_{\mu\nu}^{(\text{non-univ})}$$

These terms are expected to vanish or become negligible in the Einstein domain, but they must be retained conceptually to preserve the generality and honesty of the formulation.

7.4.8 Averaging Residuals

The final class arises from the coarse-graining operation itself. Since averaging and differentiation do not generally commute,

$$\langle \nabla_\mu Q \rangle \neq \nabla_\mu \langle Q \rangle$$

the effective continuum equations acquire residual terms associated with:

- unresolved inhomogeneity,
- finite averaging-domain structure,
- non-local contributions generated by coarse-graining.

These are collected into $\Delta_{\mu\nu}^{(\text{avg})}$.

This term is especially important conceptually, because it makes explicit that the continuum limit is approximate and controlled, not exact by definition.

7.4.9 Conservation Compatibility of the Residual Sector

Because

$$\nabla^\mu G_{\mu\nu} = 0$$

and because $T_{\mu\nu}^{\text{eff}}$ is conserved to leading order in the Einstein domain, the residual sector must satisfy corresponding consistency conditions. Specifically, any non-vanishing divergence of the leading source term must be balanced by the residual tensor, so that the full equation remains dynamically consistent. Thus, $\Delta_{\mu\nu}$ is not only a bookkeeping device; it is the tensorial structure that preserves consistency when Einstein closure is approximate rather than exact.

7.4.10 Conceptual Result

This subsection establishes an important result:

Every deviation from Einstein-domain dynamics in GETT has a specific physical origin.

Nothing is hidden in the correction tensor. The remainder is not a vague "modified gravity term," but the explicit macroscopic signature of identifiable substrate effects that become relevant when the closure assumptions weaken.

7.4.11 Summary Statement

The residual tensor $\Delta_{\mu\nu}$ contains the full non-Einstein content of the coarse-grained Φ–matter dynamics, arising from coupling modulation, higher-gradient substrate structure, non-adiabatic evolution, non-universal response, and averaging residuals, and thereby provides a controlled and physically interpretable measure of departure from Einstein-domain closure.

7.5 Emergent Gravitational Coupling (Sketch Derivation)

7.5.1 Starting Point: Portal Structure

At the level of the underlying action, the interaction between the Φ field and Standard Model matter is mediated through the Higgs sector via a density-modulated portal term of the form

$$\mathcal{L}_{\text{int}} = -\lambda_{\Phi H}(\Sigma)\, \Phi^2\, H^\dagger H$$

with

$$\lambda_{\Phi H}(\Sigma) = \lambda_0\, S(\Sigma)$$

Following electroweak symmetry breaking, $H^\dagger H \to v^2 + \delta h$, so that the leading-order coupling becomes

$$\mathcal{L}_{\text{int}} \supset -\lambda_0\, S(\Sigma)\, v^2\, \Phi^2 + \text{fluctuation terms}$$

This induces an effective coupling between the Φ field and the local matter sector through the Higgs expectation value.

7.5.2 Linear Response and Mediated Interaction

In the Einstein domain, where:

- $S(\Sigma) \approx S_0$,
- Φ variations are smooth and weak,

the Φ field may be expanded about a background configuration:

$$\Phi = \Phi_0 + \delta\Phi$$

The leading interaction between matter sources is then mediated by fluctuations $\delta\Phi$, governed by an effective Klein–Gordon operator:

$$(\Box - m_\Phi^2)\,\delta\Phi = J_{\text{eff}}$$

where the effective source term scales as

$$J_{\text{eff}} \sim \lambda_0\, S_0\, v^2 \times (\text{matter density response}).$$

Solving formally:

$$\delta\Phi \sim \frac{J_{\text{eff}}}{m_\Phi^2}$$

up to geometric factors and response coefficients.

7.5.3 Effective Interaction Strength

The induced interaction between matter components arises from the exchange of Φ fluctuations. At leading order, the effective coupling strength therefore scales as:

$$\mathcal{A} \sim \frac{(\lambda_0 S_0 v^2)^2}{m_\Phi^2} \times \mathcal{R}$$

where:

- $\mathcal{R}$ is a dimensionless response function encoding:
    - coarse-grained matter distribution,
    - averaging kernel effects (Section 4),
    - and geometric factors.

7.5.4 Identification of G_{eff}

Matching this mediated interaction to the Newtonian limit:

$$\nabla^2\Phi_{\text{grav}} = 4\pi G_{\text{eff}}\rho$$

we identify the emergent gravitational coupling as:

$$G_{\text{eff}} \sim \frac{(\lambda_0 S_0 v^2)^2}{m_\Phi^2} \times \mathcal{R}$$

Equivalently, up to order-unity factors:

$$G_{\text{eff}} \propto \frac{\lambda_0^2\, S_0^2}{m_\Phi^2} \times (\text{matter response})$$

7.5.5 Einstein-Domain Constancy

Within the Einstein domain:

- $S(\Sigma) \approx S_0$ (constant),
- $\mathcal{R}$ varies slowly under coarse-graining,
- and m_Φ is fixed,

so that:

$$G_{\mathrm{eff}} \approx \mathrm{const}$$

to leading order.

In the Einstein domain, consistency with the observed Newtonian coupling requires the response function $\mathcal{R}$ to be positive, approximately constant under coarse-graining, and of the order

$$\mathcal{R} \sim \frac{G\, m_\Phi^2}{(\lambda_0 S_0 v^2)^2}$$

up to order-unity geometric factors, so that $G_{\mathrm{eff}} \to G$ within the observational tolerance of the closure regime. This requirement constrains only the coarse-grained magnitude of $\mathcal{R}$, not its detailed microscopic form. Thus, the effective gravitational coupling appearing in the Einstein-domain field equations is not introduced by convention but arises as the macroscopic coupling strength of Φ-mediated interactions.

7.5.6 Interpretation and Scope

This derivation is schematic and establishes:

- the parametric dependence of G_{eff},
- its origin in the Φ–Higgs portal interaction,
- and its constancy within the closure regime.

A full quantum field theoretic treatment – including loop corrections, renormalisation, and precise determination of the response function $\mathcal{R}$ – is beyond the scope of the present correspondence paper and will be developed separately.

7.6 Einstein-Domain Limit

Scope: Demonstrate that, under the closure conditions established in Section 6, the residual tensor $\Delta_{\mu\nu}$ is perturbatively suppressed, so that the effective dynamical equations reduce to Einstein form at leading order.

7.6.1 Statement of the Limit

From Sections 7.3 and 7.4, the coarse-grained GETT system satisfies

$$G_{\mu\nu} = 8\pi G_{\mathrm{eff}}\, T_{\mu\nu}^{\mathrm{eff}} + \Delta_{\mu\nu}$$

with $\Delta_{\mu\nu}$ containing all non-Einstein contributions.

The Einstein-domain limit is defined as the regime in which the hypotheses of Section 6 hold throughout the averaging domain, namely:

- $S(\Sigma) \approx S_0$,

- $|\frac{\nabla S}{S}| L \ll 1$
- $|\frac{\partial_t S}{S}| \tau \ll 1$
- $|\frac{\nabla \Phi}{\Phi}| L \ll 1$
- $|\frac{\nabla\nabla \Phi}{\Phi}| L^2 \ll 1$
- $|\frac{\partial_t \Phi}{\Phi}| \tau \ll 1$
- $|\frac{\partial_t^2 \Phi}{\Phi}| \tau^2 \ll 1$
- $\epsilon_{\text{non-univ}} \ll 1$

Under these conditions, the residual tensor is perturbatively small:

$$\Delta_{\mu\nu} = O(\epsilon), \epsilon \ll 1$$

7.6.2 Suppression of the Residual Components

The decomposition of Section 7.4 shows that

$$\Delta_{\mu\nu} = \Delta_{\mu\nu}^{(S)} + \Delta_{\mu\nu}^{(\Phi\text{-grad})} + \Delta_{\mu\nu}^{(\text{adia})} + \Delta_{\mu\nu}^{(\text{non-univ})} + \Delta_{\mu\nu}^{(\text{avg})}.$$

Each contribution is suppressed by the Section 6 conditions:

- **Coupling-modulation residuals** are suppressed because $S(\Sigma)$ is locally constant to leading order.
- **Higher-gradient Φ residuals** are suppressed because the field varies smoothly across the averaging scale.
- **Non-adiabatic residuals** are suppressed because time evolution is slow relative to the coarse-graining timescale.
- **Non-universal residuals** are suppressed because matter response is effectively species-independent in the Einstein domain.
- **Averaging residuals** are suppressed because unresolved structure is small and scale separation is maintained.

Accordingly, no term in $\Delta_{\mu\nu}$ survives at leading order once the Einstein-domain conditions are imposed.

7.6.3 Leading-Order Closure

Since the residual sector is $O(\epsilon)$, the field equation becomes

$$G_{\mu\nu} = 8\pi G_{\text{eff}} T_{\mu\nu}^{\text{eff}} + O(\epsilon)$$

Thus, to leading order in the Einstein domain,

$$G_{\mu\nu} \approx 8\pi G_{\text{eff}}\, T_{\mu\nu}^{\text{eff}}$$

This is the desired Einstein-form closure. It follows that the effective metric dynamics of the coarse-grained GETT system coincide with those of General Relativity up to perturbatively controlled corrections.

7.6.4 Strict Einstein Limit

In the formal limit

$$\epsilon \to 0$$

all residual contributions vanish:

$$\Delta_{\mu\nu} \to 0$$

The dynamical equation then reduces exactly to

$$G_{\mu\nu} = 8\pi G_{\text{eff}}\, T_{\mu\nu}^{\text{eff}}$$

This establishes the Einstein field equations as the exact limiting closure of the coarse-grained Φ–matter dynamics under ideal Einstein-domain conditions.

7.6.5 Meaning of the Result

This result must be interpreted carefully. It does not mean that Einstein's equations are fundamental in GETT. Rather, it means that Einstein's equations are the unique leading-order macroscopic closure that emerges when:

- coupling is stable,
- the substrate is smooth,
- time evolution is adiabatic,
- and matter response is effectively universal.

Thus, GR is recovered not as a primitive axiom, but as a domain-limited dynamical limit of a more fundamental substrate theory.

7.6.6 Compatibility with Section 2 Requirements

The Einstein-domain limit established here satisfies the correspondence requirements of Section 2:

- the geometric structure is fully defined from the effective metric,
- the effective source tensor is well-defined,
- conservation holds to leading order,
- the curvature–source relation takes Einstein form,
- correction terms are explicitly identified and perturbatively suppressed,
- and the validity domain is clearly specified.

Accordingly, the conditions for valid Einstein-domain dynamical correspondence are met.

7.6.7 Domain-Limited Nature of the Recovery

The result remains explicitly conditional. If the Einstein-domain conditions weaken, then ϵgrows and the correction tensor $\Delta_{\mu\nu}$ becomes dynamically relevant.

Thus:

- **inside the Einstein domain:** GR is recovered,
- **outside the Einstein domain:** GETT predicts departures from Einstein dynamics.

This preserves both correspondence and falsifiability.

7.6.8 Conceptual Result

The central result of this subsection is:

Einstein's field equations arise in GETT as the leading-order closure of coarse-grained Φ-substrate dynamics when all non-Einstein contributions are suppressed by smoothness, weak coupling variation, adiabaticity, and effective universality.

7.6.9 Summary Statement

Under the Einstein-domain closure conditions, all non-Einstein contributions to the coarse-grained Φ–matter dynamics are suppressed to $O(\epsilon)$, so that the effective metric obeys Einstein-form field equations at leading order, with exact recovery in the formal limit $\epsilon \to 0$.

7.7 Emergence of G_{eff}

Scope: Identify the origin, meaning, and domain behaviour of the effective gravitational coupling G_{eff}, showing how it arises from the underlying Φ–matter interaction structure and why it is approximately constant in the Einstein domain.

7.7.1 Definition from the Closure Relation

From Section 7.5, the Einstein-domain dynamical relation takes the form

$$G_{\mu\nu} = 8\pi G_{\text{eff}}\, T_{\mu\nu}^{\text{eff}} + O(\epsilon)$$

The quantity G_{eff} is therefore defined as the proportionality constant relating curvature and effective stress-energy at leading order in the coarse-grained system.

This constant is not fundamental a priori; it is an **emergent parameter** of the macroscopic theory.

7.7.2 Origin in the Underlying Dynamics

The value of G_{eff} arises from the combined effects of:

- the baseline coupling strength λ_0,
- the modulation function $S(\Sigma)$,
- the dynamical response of the Φ substrate,
- and the coarse-graining procedure that maps microscopic dynamics to continuum variables.

Schematically, one may express

$$G_{\text{eff}} = \mathcal{G}(\lambda_0, S(\Sigma), \Phi_{\text{eff}}, \text{response properties}),$$

where $\mathcal{G}$ represents the effective coupling generated by the interaction between matter and the Φ field after averaging. This emphasises that G_{eff} is a derived quantity, not an independently specified constant.

7.7.3 Constancy in the Einstein Domain

Under the closure conditions of Section 6:

- $S(\Sigma) \approx S_0$,
- $|\nabla S|$ and $|\partial_t S|$ are negligible,
- the Φ field varies smoothly,
- and the system evolves adiabatically,

so that all local variations in the coupling structure are suppressed. In this regime:

$$G_{\text{eff}} \approx \text{const}$$

That is, although G_{eff} may depend on the underlying physical state in general, it becomes effectively constant across the averaging domain when the Einstein-domain conditions hold.

7.7.4 Stability Under Perturbations

Small perturbations in the underlying variables (e.g. small changes in Σ, Φ, or local gradients) produce only higher-order changes in G_{eff}. Formally,

$$\delta G_{\text{eff}} = O(\epsilon)$$

so that:

- leading-order dynamics are governed by a stable coupling,
- variations in G_{eff} contribute only to the residual tensor $\Delta_{\mu\nu}$.

This stability is essential for reproducing the observational success of GR in regimes where Gappears constant.

7.7.5 Relation to Standard Gravitational Constant

Within the Einstein domain, G_{eff} plays the same role as the Newtonian gravitational constant G in General Relativity.

Thus, observationally,

$$G_{\text{eff}} \to G \text{ (in the Einstein domain).}$$

This identification is not imposed, but arises because:

- the leading-order dynamics match GR,
- and the effective coupling is constant to observational precision.

7.7.6 Behaviour Outside the Einstein Domain

When the closure conditions of Section 6 are no longer satisfied:

- $S(\Sigma)$ may vary significantly,
- Φ-field gradients may become large,
- time dependence may become non-adiabatic,

and the effective coupling becomes environment-dependent. In such regimes:

$$G_{\text{eff}} = G_{\text{eff}}(x^\mu)$$

and may vary with:

- density,
- spatial position,
- or dynamical state.

These variations are encoded in the correction tensor $\Delta_{\mu\nu}$ and represent departures from Einstein-domain behaviour.

7.7.7 Conceptual Interpretation

In GETT, the gravitational coupling is not a fundamental constant of nature, but an emergent property of the interaction between matter and the Φ substrate. The significant shift from GR treating G as fundamental to GETT treating G_{eff} as **context-dependent but approximately constant in the Einstein domain**. This reframes gravity as a phenomenon whose apparent universality arises from the stability of underlying conditions, rather than from an absolute constant built into the laws of physics.

7.7.8 Role in the Overall Derivation

The identification of G_{eff} completes the curvature–source relation:

- Section 7.2 → defined curvature,
- Section 5 → defined the source,
- Section 7.3 → linked them,
- Section 7.5 → identified the coupling constant
- Section 7.6 → demonstrated suppression of residuals and leading-order closure
- Section 7.7 → identified and characterised G_eff.

At this point, the Einstein-form equation is fully specified within the GETT framework.

7.7.9 Summary Statement

The effective gravitational coupling G_{eff} emerges from the coarse-grained Φ–matter interaction structure, becomes approximately constant under Einstein-domain conditions, and reduces observationally to the standard gravitational constant while remaining fundamentally an environment-dependent quantity outside that domain. GETT explains why GR appears; Section 7 formally establishes:

$$G_{\mu\nu} = 8\pi G_{\text{eff}}\, T^{\text{eff}}_{\mu\nu} + \Delta_{\mu\nu}, \Delta_{\mu\nu} = O(\epsilon)$$

with: $\Delta_{\mu\nu} \to 0$ in the Einstein domain.

The resulting dynamical system is therefore fully consistent with the geometric and conservation structure of General Relativity, with deviations arising only through the controlled correction tensor $\Delta_{\mu\nu}$.

7.8 Section 7 Summary — Einstein-Domain Dynamics Recovery Grid

Stage	What Is Established	Key Output	Role in GR Correspondence
Metric–Substrate Link (7.1)	Effective metric defined as a functional of coarse-grained Φ–matter system	$g_{\mu\nu}^{\text{eff}} = g_{\mu\nu}^{\text{eff}}(\Phi_{\text{eff}}, S_{\text{eff}}, \dots)$	Grounds geometry in physical substrate dynamics
Curvature Construction (7.2)	Full geometric structure derived from effective metric	$R^{\rho}{}_{\sigma\mu\nu}, R_{\mu\nu}, R, G_{\mu\nu}$	Provides complete geometric side of field equations
Source Coupling (7.3)	Leading-order relation between curvature and effective source	$G_{\mu\nu} = 8\pi G_{\text{eff}} T_{\mu\nu}^{\text{eff}} + \Delta_{\mu\nu}$	Establishes Einstein-form structure
Residual Decomposition (7.4)	Explicit identification of all non-Einstein contributions	$\Delta_{\mu\nu} = \Delta^{(S)} + \Delta^{(\Phi)} + \Delta^{(\text{adia})} + \Delta^{(\text{non-univ})} + \Delta^{(\text{avg})}$	Ensures no hidden terms; full physical transparency
Einstein-Domain Limit (7.5)	Suppression of all residual terms under closure conditions	$\Delta_{\mu\nu} = O(\epsilon),\ \epsilon \ll 1$	Demonstrates recovery of Einstein equations at leading order
Exact Limit	Formal recovery of GR in ideal regime	$\Delta_{\mu\nu} \to 0 \Rightarrow G_{\mu\nu} = 8\pi G_{\text{eff}} T_{\mu\nu}^{\text{eff}}$	Establishes GR as limiting case
Emergence of G_{eff} (7.6)	Gravitational coupling derived from Φ–matter interaction structure	$G_{\text{eff}} = \mathcal{G}(\lambda_0, S(\Sigma), \Phi_{\text{eff}}, \dots)$	Replaces fundamental constant with emergent parameter
Constancy in Einstein Domain	Stability of effective coupling under closure conditions	$G_{\text{eff}} \approx \text{const}$	Matches observational behaviour of GR
Domain Dependence	Breakdown of Einstein closure outside defined regime	$\Delta_{\mu\nu} \not\ll 1$, $G_{\text{eff}} = G_{\text{eff}}(x^{\mu})$	Ensures falsifiability and predictive extension beyond GR
Closure Result	Einstein equations recovered as leading-order dynamics	$G_{\mu\nu} = 8\pi G_{\text{eff}} T_{\mu\nu}^{\text{eff}} + O(\epsilon)$	Completes dynamical correspondence

Table 7. Section 7 Summary — Einstein-Domain Dynamics Recovery Grid

Table 7 shows the step-by-step derivation of Einstein-domain dynamics from the coarse-grained Φ–matter system, demonstrating how curvature, effective source, and emergent coupling combine to yield Einstein-form field equations as a controlled leading-order closure.

8. Nature of Correction Terms ($\Delta_{\mu\nu}$)

This section characterises the correction tensor $\Delta_{\mu\nu}$ introduced in Section 7, identifying its physical origin, scaling behaviour, and role in governing departures from Einstein-domain dynamics.

The objective is to:

- classify the structure of corrections,
- quantify how and when they become significant,
- and establish their interpretation as physically meaningful deviations rather than arbitrary modifications.

This section therefore connects the formal derivation of Section 7 to **observable consequences and falsifiability**.

8.1 Origin and Classification of Corrections

Scope: Recast the residual tensor $\Delta_{\mu\nu}$in physically interpretable terms, grouping contributions by their underlying mechanism.

8.1.1 Recap of Residual Structure

From Section 7.4:

$$\Delta_{\mu\nu} = \Delta_{\mu\nu}^{(S)} + \Delta_{\mu\nu}^{(\Phi\text{-grad})} + \Delta_{\mu\nu}^{(\text{adia})} + \Delta_{\mu\nu}^{(\text{non-univ})} + \Delta_{\mu\nu}^{(\text{avg})}$$

Each component corresponds to a distinct physical effect in the Φ–matter system.

8.1.2 Coupling-Modulation Corrections

$$\Delta_{\mu\nu}^{(S)}$$

These arise when the modulation function $S(\Sigma)$ varies across the averaging domain.

Physical origin:

- spatial variation in baryonic density,
- environment-dependent coupling strength.

Effect:

- modifies effective gravitational response,
- introduces position-dependent deviations from GR,
- can mimic additional “source” terms without new matter.

8.1.3 Φ-Gradient Corrections

$$\Delta_{\mu\nu}^{(\Phi\text{-grad})}$$

These arise from higher-order spatial and temporal structure in the Φ field.

Physical origin:

- unresolved field gradients,
- local tension anisotropies,
- sub-coarse-graining structure.

Effect:

- introduces anisotropic stress,
- modifies effective pressure and curvature response,
- becomes significant in regions with strong field variation.

8.1.4 Non-Adiabatic Corrections

$$\Delta_{\mu\nu}^{(\text{adia})}$$

These arise when the system evolves too rapidly for quasi-static closure.

Physical origin:

- time-dependent changes in Φ or coupling,
- transient or dynamical processes.

Effect:

- introduces lag or memory effects,
- produces deviations from quasi-static gravitational behaviour,
- may be relevant in rapidly evolving astrophysical or cosmological systems.

8.1.5 Non-Universal Corrections

$$\Delta_{\mu\nu}^{(\text{non-univ})}$$

These arise when different forms of matter couple differently to the Φ substrate.

Physical origin:

- composition-dependent coupling,
- sector-specific interaction structure.

Effect:

- potential violations of equivalence-principle-like behaviour,
- differential response of matter species,
- suppressed in the Einstein domain but relevant in transition regimes.

8.1.6 Averaging Residual Corrections

$$\Delta_{\mu\nu}^{(\text{avg})}$$

These arise from the coarse-graining procedure itself.

Physical origin:

- non-commutativity of averaging and differentiation,
- unresolved inhomogeneity within $\mathcal{D}(x)$.

Effect:

- introduces effective non-local contributions,
- reflects limitations of the continuum approximation,
- vanishes as scale separation improves.

8.1.7 Summary Statement

The correction tensor $\Delta_{\mu\nu}$ decomposes into physically distinct contributions arising from coupling variation, substrate gradients, non-adiabatic dynamics, non-universal response, and averaging effects, each representing a controlled and interpretable deviation from Einstein-domain behaviour.

8.2 Scaling Behaviour of Corrections

Scope: Quantify the magnitude of each component of the correction tensor $\Delta_{\mu\nu}$, expressing their dependence on gradients, coupling variation, and temporal dynamics, and showing explicitly how they are controlled by the small parameter ϵ.

8.2.1 Definition of the Small Parameter

From Section 6, define a dimensionless small parameter ϵcharacterising departure from Einstein-domain conditions:

$$\epsilon \sim \max\left(\mid \frac{\nabla S}{S} \mid L, \quad \mid \frac{\partial_t S}{S} \mid \tau, \quad \mid \frac{\nabla \Phi}{\Phi} \mid L, \quad \mid \frac{\nabla\nabla \Phi}{\Phi} \mid L^2, \quad \mid \frac{\partial_t \Phi}{\Phi} \mid \tau, \quad \mid \frac{\partial_t^2 \Phi}{\Phi} \mid \tau^2, \ \epsilon_{\text{non-univ}}\right)$$

The Einstein domain is defined by:

$$\epsilon \ll 1$$

8.2.2 Scaling of Coupling-Modulation Corrections

$$\Delta_{\mu\nu}^{(S)} \sim O\left(\mid \frac{\nabla S}{S} \mid L, \quad \mid \frac{\partial_t S}{S} \mid \tau\right)$$

Thus:

- negligible when $S(\Sigma)$ is locally constant,
- grows in regions of strong density variation,
- dominates near transition regimes (e.g. threshold regions).

8.2.3 Scaling of Φ-Gradient Corrections

$$\Delta_{\mu\nu}^{(\Phi\text{-grad})} \sim O\left(\mid \frac{\nabla \Phi}{\Phi} \mid L, \quad \mid \frac{\nabla\nabla \Phi}{\Phi} \mid L^2\right)$$

Thus:

- negligible for smooth field configurations,
- increases with spatial inhomogeneity,
- becomes important in regions with strong tension gradients.

8.2.4 Scaling of Non-Adiabatic Corrections

$$\Delta_{\mu\nu}^{(\text{adia})} \sim O\left(\mid \frac{\partial_t \Phi}{\Phi} \mid \tau, \quad \mid \frac{\partial_t^2 \Phi}{\Phi} \mid \tau^2\right)$$

Thus:

- negligible in quasi-static regimes,
- grows in rapidly evolving systems,
- may contribute in transient or dynamical astrophysical environments.

8.2.5 Scaling of Non-Universal Corrections

$$\Delta_{\mu\nu}^{(\text{non-univ})} \sim O(\epsilon_{\text{non-univ}})$$

Thus:

- suppressed when matter response is effectively universal,
- grows if coupling differs between matter sectors,
- provides a potential observational discriminator.

8.2.6 Scaling of Averaging Residuals

$$\Delta_{\mu\nu}^{(\text{avg})} \sim O\left(\frac{\ell_{\text{micro}}}{L}, \quad \frac{t_{\text{micro}}}{\tau}\right)$$

Thus:

- negligible when scale separation is strong,
- increases when averaging scale approaches microscopic structure,
- reflects breakdown of continuum approximation.

8.2.7 Combined Scaling Behaviour

Collecting all contributions:

$$\Delta_{\mu\nu} = O(\epsilon)\, T_{\mu\nu}^{\text{eff}}, \epsilon \ll 1 \quad \text{(Einstein domain)}$$

Thus,

- all corrections are parametrically small relative to the leading source,
- Einstein-form dynamics dominate at leading order,
- deviations are systematically controlled.

8.2.8 Behaviour Across Regimes

- **Einstein domain:**
  $\epsilon \ll 1 \Rightarrow \Delta_{\mu\nu}$negligible
- **Transition regimes:**
  $\epsilon \sim 1 \Rightarrow \Delta_{\mu\nu}$comparable to leading term
- **Non-Einstein regimes:**
  $\epsilon \gg 1 \Rightarrow \Delta_{\mu\nu}$dominates dynamics

This provides a clear, quantitative definition of when GR holds and when it breaks down.

8.2.9 Summary Statement

Each component of the correction tensor $\Delta_{\mu\nu}$ scales with dimensionless measures of coupling variation, field gradients, temporal evolution, and scale separation, such that all corrections are $O(\epsilon)$ and therefore negligible in the Einstein domain but become significant in regimes where these controlling parameters approach unity.

8.3 Physical Interpretation of Deviations

Scope: Translate the correction tensor $\Delta_{\mu\nu}$ into physically interpretable deviations from Einstein-domain behaviour, identifying how each class of correction manifests in observable gravitational phenomena.

8.3.1 General Interpretation

From Sections 7 and 8.2, the field equation takes the form:

$$G_{\mu\nu} = 8\pi G_{\mathrm{eff}} T_{\mu\nu}^{\mathrm{eff}} + \Delta_{\mu\nu}$$

In the Einstein domain, $\Delta_{\mu\nu}$ is negligible, and standard GR behaviour is recovered.

Outside this domain, $\Delta_{\mu\nu}$ acts as an effective additional source or modification, producing observable deviations without requiring new fundamental matter components.

Thus, deviations from GR arise not from missing matter, but from additional structure in the substrate dynamics.

8.3.2 Effective Source Interpretation

At the level of phenomenology, the correction tensor can be reinterpreted as an effective additional stress-energy contribution:

$$G_{\mu\nu} = 8\pi G_{\mathrm{eff}}\left(T_{\mu\nu}^{\mathrm{eff}} + T_{\mu\nu}^{(\Delta)}\right)$$

where:

$$T_{\mu\nu}^{(\Delta)} \equiv \frac{1}{8\pi G_{\mathrm{eff}}} \Delta_{\mu\nu}$$

This allows deviations to be understood in GR-like language as:

- additional effective energy density,
- modified pressure and stress,
- or altered coupling behaviour.

However, in GETT this "extra source" is not new matter – it is the manifestation of non-Einstein substrate dynamics.

8.3.3 Low-Density Regime (Dark-Matter-Like Behaviour)

In regions of low baryonic density:

- $S(\Sigma)$ varies significantly,
- coupling-modulation terms become important,
- gradient structure in the Φ field becomes non-negligible.

This leads to:

- enhanced effective gravitational response,
- deviations from Newtonian/GR expectations,
- apparent additional mass contributions.

Phenomenologically, this reproduces:

- flat galaxy rotation curves,
- excess gravitational lensing relative to visible matter,
- mass discrepancies in low-density environments.

In GETT, these effects arise from:

$$\Delta_{\mu\nu}^{(S)} + \Delta_{\mu\nu}^{(\Phi\text{-grad})}$$

rather than from non-baryonic dark matter.

8.3.4 Transition Regimes (Threshold Behaviour)

Near density thresholds (e.g. the characteristic density identified in prior GETT work):

- $\left|\frac{dS}{d\Sigma}\right|$ becomes significant,
- coupling transitions between regimes,
- corrections change rapidly with environment.

This produces:

- sharp changes in dynamical behaviour,
- regime-dependent gravitational response,
- observable transitions in kinematic relations.

These features correspond to:

- onset of dark-matter-like behaviour in galaxies,
- regime changes in stellar velocity dispersion (e.g. Gaia results),
- environment-dependent deviations from GR.

8.3.5 High-Density Regime (GR Recovery and Beyond)

In high-density environments:

- $S(\Sigma)$ approaches a stable plateau,
- coupling variation is minimal,
- Φ-field gradients are suppressed.

This yields:

- recovery of GR behaviour,
- stable effective coupling $G_{\text{eff}} \approx \text{const}$,
- negligible correction tensor.

This is consistent with:

- Solar System tests of GR,
- binary pulsar observations,
- strong-field tests where deviations are tightly constrained.

At ultra-high densities, additional effects (e.g. symmetry restoration) may arise, but these lie beyond the Einstein domain and are not required for GR correspondence.

8.3.6 Dynamical Regimes (Non-Adiabatic Effects)

In systems with rapid evolution:

- mergers,
- strong time-dependent gravitational fields,
- early-universe dynamics,

non-adiabatic corrections become relevant:

$$\Delta_{\mu\nu}^{(\text{adia})}$$

These may produce:

- transient deviations from GR predictions,
- time-dependent effective coupling,
- modified propagation or damping of gravitational disturbances.

Such effects provide potential observational tests in:

- gravitational wave signals,
- rapidly evolving astrophysical systems.

8.3.7 Equivalence Principle Considerations

Non-universal corrections:

$$\Delta_{\mu\nu}^{(\text{non-univ})}$$

introduce the possibility that:

- different matter components experience slightly different effective coupling,
- the equivalence principle is not strictly universal.

In the Einstein domain:

- these effects are suppressed,
- GR-like universality is recovered.

Outside the domain:

- small deviations may appear,
- offering a potential experimental signature.

8.3.8 Continuum Breakdown (Averaging Limits)

When scale separation fails:

$$\Delta_{\mu\nu}^{(\text{avg})}$$

becomes significant, leading to:

- breakdown of smooth continuum description,
- effective non-local behaviour,
- limitations of geometric representation.

This regime highlights that GR is not universally valid but depends on the existence of a well-defined continuum limit.

8.3.9 Unified Interpretation

Across all regimes, deviations from GR can be understood as arising from a single principle:

departures from Einstein-domain behaviour occur when the Φ–matter system fails to satisfy the conditions required for smooth, universal, adiabatic coarse-grained closure.

Thus:

- GR is the stable plateau,
- deviations are structured and predictable,
- and all modifications trace back to identifiable physical causes.

8.3.10 Summary Statement

The correction tensor $\Delta_{\mu\nu}$ manifests observationally as effective additional source terms, modified coupling behaviour, or transient dynamical effects, with distinct signatures across low-density, transitional, high-density, and non-adiabatic regimes, providing a unified and physically grounded explanation of deviations from Einstein-domain gravitational dynamics.

8.4 Observational Signatures and Testability

Scope: Identify concrete, falsifiable observational signatures associated with the correction tensor $\Delta_{\mu\nu}$, and define how GETT can be empirically distinguished from General Relativity and alternative theories.

8.4.1 Principle of Testability

From Sections 7 and 8:

$$G_{\mu\nu} = 8\pi G_{\text{eff}}\, T_{\mu\nu}^{\text{eff}} + \Delta_{\mu\nu}$$

In the Einstein domain, $\Delta_{\mu\nu} \approx 0$, and GR is recovered. Testability therefore rests on identifying regimes where:

- $\epsilon \ll/1$,
- $\Delta_{\mu\nu}$becomes measurable,
- and predictions diverge from GR.

Thus, GETT is falsifiable through environment-dependent deviations tied to density, gradients, and dynamics.

8.4.2 Density-Dependent Gravitational Response

A central prediction is that gravitational behaviour depends directly on baryonic density Σ.

Prediction:

- Systems with similar mass but different density distributions exhibit different effective gravitational behaviour.

Observable tests:

- galaxy rotation curves as a function of baryonic density profile,
- scaling relations across galaxies (e.g. SPARC-like datasets),
- vertical stellar kinematics (e.g. Gaia DR3 analyses).

Falsification criterion:

- absence of a consistent density-dependent transition in gravitational behaviour.

8.4.3 Threshold Behaviour

GETT predicts the existence of characteristic density thresholds where gravitational behaviour changes regime.

Prediction:

- sharp or smooth transitions in dynamical behaviour near specific density scales.

Observable tests:

- onset of flat rotation curves,
- changes in velocity dispersion profiles,
- environmental transitions in galaxy dynamics.

Falsification criterion:

- no statistically significant or reproducible threshold behaviour across independent systems.

8.4.4 Absence of Non-Baryonic Dark Matter Requirement

GETT predicts that apparent dark-matter effects arise from $\Delta_{\mu\nu}$, not unseen matter.

Prediction:

- gravitational anomalies correlate with baryonic density and structure,
- no need for independent dark matter distribution.

Observable tests:

- correlation strength between baryonic mass distribution and inferred gravitational field,
- residuals in mass–rotation relations,
- gravitational lensing consistent with density-modulated dynamics.

Falsification criterion:

- detection of gravitational effects that cannot be explained by baryonic density-dependent dynamics.

8.4.5 Effective Variation of Gravitational Coupling

Outside the Einstein domain:

$$G_{\text{eff}} = G_{\text{eff}}(x^{\mu})$$

Prediction:

- small but measurable variations in effective gravitational strength across environments.

Observable tests:

- precision measurements of gravitational coupling in different density regimes,

- astrophysical systems with varying environments,
- cosmological-scale consistency checks.

Falsification criterion:

- strict universality of G across all regimes to arbitrarily high precision.

8.4.6 Deviations in Strong Gradient or Low-Density Regimes

Where $\Delta_{\mu\nu}^{(\Phi\text{-grad})}$ becomes significant:

Prediction:

- deviations from GR in outer galaxy regions,
- enhanced gravitational effects without additional mass,
- anisotropic stress signatures.

Observable tests:

- outer rotation curve shapes,
- weak gravitational lensing profiles,
- structure formation in low-density environments.

Falsification criterion:

- precise agreement with GR predictions in all low-density regimes without additional matter.

8.4.7 Non-Adiabatic and Dynamical Effects

Where $\Delta_{\mu\nu}^{(\text{adia})}$ is relevant:

Prediction:

- transient deviations from GR during rapid evolution,
- possible modification of gravitational wave propagation or damping.

Observable tests:

- gravitational wave signals (timing, amplitude evolution),
- merger dynamics,
- early-universe cosmological evolution.

Falsification criterion:

- complete agreement with GR predictions in all dynamical regimes.

8.4.8 Equivalence Principle Tests

Non-universal coupling effects predict:

- small deviations from perfect equivalence-principle behaviour outside the Einstein domain.

Observable tests:

- precision experiments comparing acceleration of different materials,
- astrophysical tests of composition-dependent motion.

Falsification criterion:

- strict universality across all regimes with no detectable deviation.

8.4.9 Continuum Limit Breakdown

If coarse-graining fails:

Prediction:

- deviations from smooth geometric behaviour,
- effective non-local gravitational effects.

Observable tests:

- small-scale structure anomalies,
- deviations from standard continuum gravitational predictions.

Falsification criterion:

- universal validity of continuum GR at all scales.

8.4.10 Unified Testability Statement

All observable deviations predicted by GETT arise from a single principle:

gravitational behaviour depends on the local state of the Φ–matter system, as encoded by density, gradients, and dynamical evolution.

Thus:

- deviations are not arbitrary,
- they are structured and parameter-controlled,
- and they are tied to measurable physical quantities.

8.4.11 Summary Statement

GETT is empirically testable through density-dependent gravitational behaviour, threshold transitions, environment-dependent coupling, and controlled deviations in low-density, high-gradient, and dynamical regimes, providing clear falsification criteria distinct from both GR and dark-matter-based models.

Component	Physical Origin	Scaling Behaviour	Observable Effect	Einstein-Domain Behaviour
Coupling Modulation $\Delta_{\mu\nu}^{(S)}$	Variation of the modulation function $S(\Sigma)$with baryonic density.	$\Delta_{\mu\nu}^{(S)} \sim O\left(\mid \frac{\nabla S}{S} \mid L, \ \mid \frac{\partial_t S}{S} \mid \tau \right)$	Density-dependent gravitational strength; dark-matter-like effects.	Suppressed when $S(\Sigma) \approx$ const.
Φ-Gradient Structure $\Delta_{\mu\nu}^{(\Phi\text{-grad})}$	Spatial and temporal inhomogeneity of Φ field	$\Delta_{\mu\nu}^{(\Phi\text{-grad})} \sim O\left(\mid \frac{\nabla \Phi}{\Phi} \mid L, \ \mid \frac{\nabla\nabla \Phi}{\Phi} \mid L^2 \right)$	Anisotropic stress; enhanced outer-region gravity; lensing deviations.	Negligible for smooth field configurations.
Non-Adiabatic Dynamics $\Delta_{\mu\nu}^{(\text{adia})}$	Rapid time evolution of Φ or coupling	$\Delta_{\mu\nu}^{(\text{adia})} \sim O\left(\mid \frac{\partial_t \Phi}{\Phi} \mid \tau, \ \mid \frac{\partial_t^2 \Phi}{\Phi} \mid \tau^2 \right)$	Transient deviations; modified dynamical behaviour; potential gravitational-wave signatures.	Negligible under slow evolution.
Non-Universal Coupling $\Delta_{\mu\nu}^{(\text{non-univ})}$	Composition-dependent interaction	$O(\epsilon_{\text{non-univ}})$	Small equivalence-principle deviations; species-dependent response	Suppressed to observational limits
Averaging Residuals $\Delta_{\mu\nu}^{(\text{avg})}$	Finite coarse-graining and non-commutativity	$O\left(\frac{\ell_{\text{micro}}}{L}, \ \frac{t_{\text{micro}}}{\tau} \right)$	Non-local or small-scale deviations; breakdown of continuum approximation	Negligible with strong scale separation
Effective Source Interpretation	Recasting $\Delta_{\mu\nu}$ as $T_{\mu\nu}^{(\Delta)}$	$O(\epsilon)\, T_{\mu\nu}^{\text{eff}}$	Apparent additional mass/energy without new matter	Vanishes at leading order
Low-Density Regime	Strong coupling variation + gradients	$\epsilon \sim 1$	Flat rotation curves; lensing excess; galaxy-scale anomalies	Outside Einstein domain
Threshold Behaviour	Rapid change in $S(\Sigma)$	Localised increase in ϵ	Regime transitions in dynamics (e.g. velocity dispersion shifts)	Boundary of Einstein domain
High-Density Regime	Stable coupling plateau	$\epsilon \ll 1$	GR recovery; constant G_{eff}	Einstein domain
Testability Framework	All corrections tied to physical parameters	Controlled by ϵ	Predictive, falsifiable deviations from GR	Fully consistent with GR within domain

Table 8. Summary – Correction Terms and Observational Consequences Grid

Table 8 shows the physical origin, scaling behaviour, and observable consequences of the correction tensor $\Delta_{\mu\nu}$, demonstrating how all deviations from Einstein-domain dynamics arise from identifiable and parameter-controlled features of the Φ–matter system.

9. Recovery of Known Limits

This section demonstrates that the Einstein-domain dynamics derived in Section 7 reproduce the established gravitational limits of classical and relativistic physics.

The objective is to show that:

- Newtonian gravity is recovered in the weak-field, non-relativistic limit,
- previously established GETT correspondence results (Paper 2) are retained,
- and standard relativistic solutions emerge consistently within the Einstein domain.

This ensures that GETT does not replace successful theories but reconstructs them as domain-limited approximations.

9.1 Weak-Field (Newtonian) Limit

Scope: Demonstrate that the Einstein-domain equations reduce to the Poisson equation for gravity in the weak-field, low-velocity limit.

9.1.1 Assumptions

Consider the standard weak-field conditions:

- gravitational fields are small,
- spacetime deviations from flatness are weak,
- velocities satisfy $v \ll c$,
- time derivatives are negligible compared to spatial variation.

In this regime, the effective metric may be written as:

$$g_{\mu\nu}^{\text{eff}} = \eta_{\mu\nu} + h_{\mu\nu}, | h_{\mu\nu} | \ll 1$$

9.1.2 Einstein-Domain Field Equation

From Section 7, in the Einstein domain:

$$G_{\mu\nu} = 8\pi G_{\text{eff}} T_{\mu\nu}^{\text{eff}}$$

In the weak-field limit, the dominant component is the time-time equation:

$$G_{00} \approx 8\pi G_{\text{eff}} T_{00}^{\text{eff}}$$

9.1.3 Identification of Gravitational Potential

In this limit, the metric perturbation is related to the Newtonian gravitational potential Φ_{grav}via:

$$h_{00} \approx -2\Phi_{\text{grav}}$$

The Einstein tensor reduces to:

$$G_{00} \approx 2\nabla^2\Phi_{\text{grav}}$$

9.1.4 Recovery of Poisson Equation

Substituting into the field equation:

$$2\nabla^2\Phi_{\text{grav}} = 8\pi G_{\text{eff}}\,\rho_{\text{eff}},$$

which simplifies to:

$$\nabla^2\Phi_{\text{grav}} = 4\pi G_{\text{eff}}\,\rho_{\text{eff}}.$$

This is the Poisson equation for Newtonian gravity.

9.1.5 Interpretation in GETT

This result shows that:

- Newtonian gravity emerges as the weak-field limit of the Einstein-domain equations,
- the effective gravitational constant reduces to $G_{\text{eff}} \approx G$,
- and the source term is the effective energy density derived from the Φ–matter system.

Thus, the classical limit is fully recovered.

9.1.6 Summary Statement

In the weak-field, non-relativistic limit, the Einstein-domain equations of GETT reduce to the Poisson equation [8], recovering Newtonian gravity as the leading-order approximation.

9.2 Consistency with Paper 2 (Newtonian Correspondence)

Scope: Demonstrate that the Newtonian limit derived here from Einstein-domain dynamics is fully consistent with, and directly recovers, the results established independently in Paper 2 of the GETT Correspondence Series.

9.2.1 Recap of Paper 2 Result

In Paper 2, Newtonian gravity was derived directly from the Φ-substrate dynamics in the mid-density regime, yielding:

$$\nabla^2\Phi_{\text{grav}} = 4\pi G\rho$$

with:

- Φ_{grav} interpreted as the macroscopic potential arising from Φ-field tension gradients,
- G emerging as an effective constant under stable coupling conditions,
- gravitational acceleration given by

$$\mathbf{a} = -\nabla\Phi_{\text{grav}}$$

This result was obtained without invoking spacetime curvature, purely from substrate dynamics.

9.2.2 Result from the Present Work

In Section 9.1 of the present paper, the weak-field limit of the Einstein-domain equations yielded:

$$\nabla^2\Phi_{\text{grav}} = 4\pi G_{\text{eff}}\,\rho_{\text{eff}}$$

Under Einstein-domain conditions:

- $G_{\text{eff}} \approx G$,
- $\rho_{\text{eff}} \to \rho$at leading order,
- correction terms are suppressed.

Thus, the same Poisson equation is recovered.

9.2.3 Equivalence of the Two Derivations

The key result is that:

- **Paper 2:** derives Newtonian gravity directly from Φ-substrate dynamics,
- **Paper 4.2:** derives Einstein-domain dynamics from the same substrate and then recovers Newtonian gravity as a limiting case.

Both paths yield the same governing equation:

$$\nabla^2 \Phi_{\text{grav}} = 4\pi G\rho.$$

This establishes internal consistency across the correspondence series.

9.2.4 Interpretation

This equivalence demonstrates that:

- the Newtonian limit is not an independent assumption,
- it is a consistent consequence of the same underlying physical mechanism,
- and it can be obtained either:
  - directly from substrate dynamics (Paper 2), or
  - via Einstein-domain closure followed by weak-field reduction (Paper 4.2).

Thus, Newtonian gravity is a robust, multi-path emergent limit within GETT.

9.2.5 Conceptual Significance

This result is important for two reasons:

1. **Consistency Across Domains**
   The same physical mechanism produces consistent results across different derivation routes.
2. **Hierarchy of Emergence**
   The structure is layered:
   - Φ-substrate dynamics (fundamental)
     → Einstein-domain geometry (Section 7)
     → Newtonian limit (Section 9.1)

This confirms that Newtonian gravity is not only recoverable but is embedded consistently within the full dynamical framework.

9.2.6 Summary Statement

The Newtonian limit derived from Einstein-domain dynamics is identical to that obtained independently in Paper 2 from Φ-substrate dynamics, confirming that GETT consistently reproduces classical gravity across both direct and geometry-mediated derivation pathways.

9.3 Static Symmetric Solutions (Schwarzschild-like Behaviour)

Scope: Establish that, within the Einstein domain, the effective metric equations admit the standard static, spherically symmetric vacuum structure associated with Schwarzschild-type behaviour, without requiring a full exact solution derivation in the present paper.

9.3.1 Static Vacuum Regime

Consider a region exterior to a compact, approximately static, spherically symmetric source.

In this regime:

- matter support is confined to an interior region,
- the exterior effective source is negligible to leading order,
- the Einstein-domain conditions remain satisfied,
- coupling is effectively constant,
- the Φ substrate varies smoothly and adiabatically.

Accordingly, outside the source:

$$T_{\mu\nu}^{\text{eff}} \approx 0, \Delta_{\mu\nu} = O(\epsilon), \epsilon \ll 1$$

The Einstein-domain field equation therefore reduces to:

$$G_{\mu\nu} = O(\epsilon)$$

In the strict Einstein-domain limit $\epsilon \to 0$, this becomes the vacuum Einstein equation:

$$G_{\mu\nu} = 0$$

9.3.2 General Static Spherically Symmetric Metric Form

The most general static, spherically symmetric line element may be written as

$$ds^2 = -e^{2\alpha(r)}dt^2 + e^{2\beta(r)}dr^2 + r^2 d\Omega^2$$

where:

- $\alpha(r)$ and $\beta(r)$ are radial functions,
- $d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$.

Since the Einstein-domain equations reduce to vacuum Einstein form at leading order, the functions $\alpha(r)$ and $\beta(r)$ satisfy the same leading-order vacuum constraints as in GR.

9.3.3 Schwarzschild-Type Exterior Behaviour

It follows that, to leading order in the Einstein domain, the exterior solution takes the standard Schwarzschild form:

$$ds^2 = -\left(1 - \frac{2G_{\text{eff}}M}{r}\right)dt^2 + \left(1 - \frac{2G_{\text{eff}}M}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

where M is the effective gravitating mass of the source. Thus, the familiar static exterior behaviour of GR is recovered, with the replacement:

$$G \to G_{\text{eff}}$$

which reduces observationally to the standard gravitational constant in the Einstein domain.

9.3.4 Interpretation in GETT

In GR, the Schwarzschild solution is interpreted as the vacuum curvature generated by a central mass. In GETT, the same metric structure is reinterpreted as the effective geometric encoding of the coarse-grained Φ-substrate configuration surrounding a static spherical source.

That is:

- the underlying ontology remains substrate-based,
- the exterior geometry is emergent,
- but the leading observable metric structure is the same.

Thus, Schwarzschild-like behaviour is recovered as an Einstein-domain limit of the Φ–matter dynamics.

9.3.5 Role of Residual Corrections

Outside the strict Einstein-domain limit, residual corrections remain:

$$G_{\mu\nu} = \Delta_{\mu\nu}, \Delta_{\mu\nu} = O(\epsilon)$$

These corrections may induce:

- small environment-dependent deviations from exact Schwarzschild form,
- weak radial dependence in G_{eff},
- higher-order structure in low-density or transition regimes.

However, within the regime relevant to classical Solar System and compact-object exterior tests, these corrections are suppressed by the same closure conditions that recover GR more generally.

9.3.6 Scope of the Present Claim

The present paper does not claim to provide a full exact classification of static spherically symmetric solutions in GETT, nor to solve all possible vacuum or near-vacuum configurations beyond the Einstein domain.

What is established here is narrower and sufficient:

- the Einstein-domain equations reduce to the standard vacuum Einstein structure,
- the leading static symmetric exterior metric is Schwarzschild-like,
- and any deviations are perturbative and controlled by $\Delta_{\mu\nu}$.

A fuller treatment of exact solutions and beyond-Einstein-domain departures belongs to later work.

9.3.7 Summary Statement

Within the Einstein domain, the coarse-grained GETT equations admit the same leading-order static, spherically symmetric vacuum structure as General Relativity, recovering Schwarzschild-like exterior behaviour while reinterpreting that geometry as the emergent macroscopic expression of the surrounding Φ-substrate configuration.

Limit / Regime	Starting Point in GETT	Recovered Result	Conditions Required	Role in Correspondence
Weak-Field Limit (9.1)	Einstein-domain equation $G_{\mu\nu} = 8\pi G_{\mathrm{eff}} T_{\mu\nu}^{\mathrm{eff}}$ with small metric perturbations	Poisson equation $\nabla^2 \Phi_{\mathrm{grav}} = 4\pi G_{\mathrm{eff}} \rho_{\mathrm{eff}}$	Weak fields, $v \ll c$, smooth Φ, adiabatic regime	Recovers Newtonian gravity from Einstein-domain dynamics
Newtonian Correspondence (9.2)	Direct Φ-substrate dynamics (Paper 2) and Einstein-domain reduction (this work)	Identical Poisson equation $\nabla^2 \Phi_{\mathrm{grav}} = 4\pi G \rho$	Mid-density regime, stable coupling, negligible corrections	Demonstrates internal consistency across GETT correspondence series
Static Vacuum Regime (9.3)	Einstein-domain limit with $T_{\mu\nu}^{\mathrm{eff}} \approx 0$ and $\Delta_{\mu\nu} \ll 1$	Vacuum equation $G_{\mu\nu} = 0$	Smooth Φ field, constant coupling, negligible residuals	Establishes correct vacuum limit of the theory
Spherically Symmetric Solutions (9.3)	Static, spherically symmetric metric ansatz in Einstein domain	Schwarzschild-like metric $ds^2 = -(1 - 2G_{\mathrm{eff}} M/r)dt^2 + \cdots$	Einstein-domain conditions, weak residual corrections	Recovers standard exterior gravitational solution
Gravitational Coupling	Emergent G_{eff} from Φ–matter interaction	$G_{\mathrm{eff}} \rightarrow G$ (observationally)	$S(\Sigma) \approx$ const, weak variation	Matches observed constancy of gravitational constant
Residual Behaviour	Correction tensor $\Delta_{\mu\nu}$	$\Delta_{\mu\nu} = O(\epsilon)$	$\epsilon \ll 1$	Ensures deviations are controlled and negligible in known limits
Hierarchy of Limits	Φ-substrate → Einstein-domain → weak-field reduction	Consistent multi-level emergence	Valid closure conditions at each level	Confirms layered structure of physical laws in GETT

Table 9. Section 9 Summary — Recovery of Known Limits Grid

Table 9 shows the recovery of established gravitational limits within GETT, demonstrating that Newtonian gravity, vacuum solutions, and Schwarzschild-like behaviour all emerge consistently from the Einstein-domain dynamics under the appropriate conditions.

10. Domain of Validity

This section defines the precise physical and mathematical conditions under which the Einstein-domain dynamical correspondence established in Section 7 is valid.

The objective is to:

- formalise the domain in which the effective metric obeys Einstein-form equations,
- express the closure conditions as explicit inequalities,
- and define the boundaries beyond which deviations from General Relativity must occur.

This establishes the theory as domain-limited, controlled, and falsifiable.

10.1 Einstein-Domain Definition

Scope: Provide a formal definition of the regime in which Einstein-form dynamics are recovered.

10.1.1 Einstein-Domain Conditions

The Einstein domain is defined as the set of spacetime regions in which the following conditions hold over the coarse-graining scales L and τ:

(A) Mid-Density Regime (Stable Coupling)

$$S(\Sigma) \approx S_0, \left| \frac{dS}{d\Sigma} \right| \approx 0$$

(B) Weak Coupling Variation

$$\left| \frac{\nabla S}{S} \right| L \ll 1, \left| \frac{\partial_t S}{S} \right| \tau \ll 1$$

(C) Smooth Φ-Field Structure

$$\left| \frac{\nabla \Phi}{\Phi} \right| L \ll 1, \left| \frac{\nabla\nabla \Phi}{\Phi} \right| L^2 \ll 1$$

(D) Adiabatic Evolution

$$\left| \frac{\partial_t \Phi}{\Phi} \right| \tau \ll 1, \left| \frac{\partial_t^2 \Phi}{\Phi} \right| \tau^2 \ll 1$$

(E) Effective Universality

$$\epsilon_{\text{non-univ}} \ll 1$$

10.1.2 Compact Definition

Define the dimensionless parameter:

$$\epsilon = \max\left(\left| \frac{\nabla S}{S} \right| L, \ \left| \frac{\partial_t S}{S} \right| \tau, \ \left| \frac{\nabla \Phi}{\Phi} \right| L, \ \left| \frac{\nabla\nabla \Phi}{\Phi} \right| L^2, \ \left| \frac{\partial_t \Phi}{\Phi} \right| \tau, \ \left| \frac{\partial_t^2 \Phi}{\Phi} \right| \tau^2, \ \epsilon_{\text{non-univ}} \right)$$

Then the Einstein domain is defined by:

$$\epsilon \ll 1$$

10.1.3 Resulting Dynamics

Within this domain:

$$G_{\mu\nu} = 8\pi G_{\text{eff}} T_{\mu\nu}^{\text{eff}} + O(\epsilon)$$

with:

$$G_{\text{eff}} \approx \text{const.}$$

Thus, Einstein-form dynamics hold to leading order.

10.2 Domain Boundaries and Breakdown

Scope: Define when and why the Einstein-domain approximation fails.

10.2.1 Breakdown Conditions

The Einstein-domain description fails when any of the following occur:

- $\mid \frac{\nabla S}{S} \mid L \sim 1$or larger
- $\mid \frac{\nabla \Phi}{\Phi} \mid L \sim 1$
- $\mid \frac{\partial_t \Phi}{\Phi} \mid \tau \sim 1$
- $\epsilon_{\text{non-univ}} \not\ll 1$
- scale separation fails ($L \sim \ell_{\text{micro}}$)

10.2.2 Consequences of Breakdown

When these conditions are violated:

- $\Delta_{\mu\nu}$ becomes comparable to the leading term,
- G_{eff} may vary significantly,
- local tensorial closure may weaken or fail,
- GR is no longer a valid approximation.

Thus, when these conditions are violated, the correction tensor becomes comparable to the leading Einstein-domain source term:

$$\Delta_{\mu\nu} \sim 8\pi G_{\text{eff}} T_{\mu\nu}^{\text{eff}}$$

10.2.3 Physical Interpretation

Breakdown corresponds physically to:

- strong density variation (low-density regimes),
- large Φ-field gradients (outer galactic regions),
- rapid dynamical evolution,
- or breakdown of universality.

These are precisely the regimes where GETT predicts deviations from GR.

10.3 Domain Hierarchy

Scope: Place the Einstein domain within the broader structure of GETT regimes.

10.3.1 Hierarchical Structure

The full theory admits a hierarchy of regimes:

1. **Einstein Domain**
   $\epsilon \ll 1$ → GR recovered

2. **Transition Regime**
   $\epsilon \sim 1$ → corrections comparable to leading order

3. **Non-Einstein Regime**
   $\epsilon \gg 1$ → modified dynamics dominate

10.3.2 Interpretation

This hierarchy shows that:

- GR is a **special case**, not a universal theory,
- deviations are continuous and controlled,
- and the theory naturally explains both agreement with GR and observed anomalies.

10.4 Formal Statement of Validity

Scope: Provide a concise, publication-ready statement.

10.4.1 Formal Statement

The Einstein-domain dynamical correspondence derived in Section 7 is valid in spacetime regions where the Φ–matter system satisfies the closure conditions defined above, such that the dimensionless deviation parameter $\epsilon \ll 1$.

In this regime:

- the effective metric admits a smooth continuum description,
- the effective stress-energy tensor is conserved to leading order,
- and the dynamical equations reduce to Einstein form with perturbatively controlled corrections.

10.4.2 Limitation Statement

Outside this domain, the assumptions required for Einstein closure fail, and the full dynamics of the Φ–matter system must be retained, resulting in departures from General Relativity.

10.5 Summary Grid

Table 10 shows the defining conditions of the Einstein domain, summarising the constraints required for the effective dynamics to reduce to Einstein form.

Component	Condition	Interpretation	Domain Implication
Coupling stability	$S(\Sigma) \approx S_0$, $dS/d\Sigma \approx 0$	Mid-density regime with negligible coupling variation	Einstein dynamics valid; no density-driven corrections
Spatial smoothness	$(\nabla\Phi / \Phi) \cdot L \ll 1$	Field varies slowly across coarse-graining scale	Gradient corrections suppressed; continuum valid
Adiabatic evolution	$(u\cdot\nabla\Phi / \Phi) \cdot \tau \ll 1$	Temporal evolution is slow	No leading-order time-dependent corrections
Scale separation	$\ell\_micro / L \ll 1$	Microscopic and macroscopic scales are well separated	Coarse-graining valid; averaging residuals negligible
Closure condition	$\varepsilon \ll 1$	All constraints simultaneously satisfied	$G_{\mu\nu} = 8\pi G_{\text{eff}} T_{\mu\nu}^{\text{eff}}$ (Einstein domain)

Table 10. Domain of Validity for Einstein-Domain Dynamics

10.6 Summary Statement

The Einstein-domain behaviour of GETT is valid only within a precisely defined regime characterised by smooth Φ-field structure, weak coupling variation, adiabatic evolution, and effective universality, with all deviations from GR controlled by a small parameter ϵ, thereby establishing the theory as domain-limited and explicitly falsifiable.

11. Failure of Einstein Closure (Physical Interpretation)

This section interprets the breakdown of Einstein-domain dynamics in physical terms, identifying the mechanisms by which the closure conditions defined in Section 10 fail, and explaining the resulting departure from General Relativity.

The objective is to show that:

- GR does not fail arbitrarily,
- its breakdown follows directly from the violation of well-defined physical conditions,
- and the resulting deviations are structured, predictable, and physically interpretable within GETT.

11.1 Nature of Einstein Closure

Scope: Clarify what is meant by "Einstein closure" and why it is conditional.

11.1.1 Definition of Closure

Einstein closure refers to the regime in which the coarse-grained Φ–matter dynamics admit a local, tensorial relation of the form:

$$G_{\mu\nu} = 8\pi G_{\text{eff}}\, T_{\mu\nu}^{\text{eff}}$$

with:

- a smooth effective metric,
- a conserved effective stress-energy tensor,
- and a stable coupling constant.

This closure is not fundamental; it is a leading-order truncation of a more general dynamical system.

11.1.2 Conditions for Closure

As established in Section 10, closure requires:

- smooth Φ-field structure,
- weak coupling variation,
- adiabatic evolution,
- effective universality,
- and strong scale separation.

Only when these conditions hold does the system admit Einstein-form dynamics.

11.1.3 Interpretation

Einstein closure is therefore a **regime-dependent simplification**, not a universal law.

GR emerges when the underlying system is sufficiently smooth, stable, and uniform.

11.2 Mechanisms of Breakdown

Scope: Identify the physical mechanisms that cause closure to fail.

11.2.1 Coupling Variation Breakdown

When:

$$|\frac{\nabla S}{S}|\, L \sim 1$$

the assumption of constant coupling fails.

Consequence:

- G_{eff} becomes environment-dependent,
- curvature–source relation acquires additional terms,
- Einstein-form closure is no longer valid.

11.2.2 Gradient Breakdown

When:

$$|\frac{\nabla\Phi}{\Phi}|L \sim 1$$

the assumption of smooth substrate structure fails.

Consequence:

- higher-order gradient terms become significant,
- anisotropic stress appears,
- geometric description becomes incomplete at leading order.

11.2.3 Non-Adiabatic Breakdown

When:

$$|\frac{\partial_t\Phi}{\Phi}|\tau \sim 1$$

the system evolves too rapidly for quasi-static closure.

Consequence:

- time-dependent corrections appear,
- delayed or memory effects emerge,
- instantaneous Einstein-form dynamics break down.

11.2.4 Non-Universality Breakdown

When:

$$\epsilon_{\text{non-univ}} \ll\!\!/ 1$$

different matter components respond differently.

Consequence:

- equivalence-principle-like behaviour is violated,
- no single universal metric description suffices,
- Einstein closure fails at the level of universality.

11.2.5 Averaging Breakdown

When:

$$L \sim \ell_{\text{micro}}$$

scale separation fails.

Consequence:

- continuum approximation breaks down,
- effective geometric description becomes invalid,
- non-local effects emerge.

11.3 Consequences of Breakdown

Scope: Describe what replaces Einstein dynamics when closure fails.

11.3.1 Growth of Correction Tensor

As closure conditions fail:

$$\Delta_{\mu\nu} \ll / 8\pi G_{\text{eff}} T_{\mu\nu}^{\text{eff}}$$

Thus:

- corrections become dynamically significant,
- Einstein-form equations are no longer a good approximation.

11.3.2 Emergence of Modified Dynamics

The full dynamics are then governed by:

$$G_{\mu\nu} = 8\pi G_{\text{eff}}\, T_{\mu\nu}^{\text{eff}} + \Delta_{\mu\nu}$$

with $\Delta_{\mu\nu}$ no longer negligible.

This produces:

- environment-dependent gravitational behaviour,
- deviations from GR predictions,
- new observable phenomena.

11.3.3 Regime-Specific Behaviour

Different breakdown mechanisms dominate in different regimes:

- **low-density regimes:** coupling modulation dominates,
- **high-gradient regions:** Φ-gradient terms dominate,
- **dynamical systems:** non-adiabatic terms dominate,
- **composition-sensitive systems:** non-universal terms dominate.

Thus, deviations are **structured, not arbitrary**.

11.4 Conceptual Interpretation

Scope: Provide the key philosophical insight.

11.4.1 Reframing General Relativity

In GETT:

- GR is not incorrect,
- it is not incomplete in its domain,
- but it is not universal.

It is the leading-order effective theory of a deeper physical system.

11.4.2 Core Insight

The central insight is:

General Relativity describes the behaviour of gravity only when the Φ–matter system satisfies the conditions required for Einstein closure.

When those conditions fail, GR must fail.

11.4.3 Nature of Physical Law

This leads to a broader conclusion:

- physical laws may be **domain-limited approximations**,
- rather than universally exact descriptions,
- emerging from deeper underlying dynamics.

11.5 Summary Statement

Einstein-domain closure fails when the physical conditions required for smooth, stable, and universal coarse-grained Φ–matter dynamics are violated, leading to controlled and physically interpretable deviations from General Relativity, thereby establishing GR as a domain-limited effective theory rather than a fundamental universal law.

12. Correspondence Audit Table

This section provides a formal audit of the correspondence between GETT and General Relativity within the Einstein domain, evaluating whether the structural, dynamical, and consistency requirements defined in Section 2 have been satisfied.

The objective is to:

- present a clear, itemised verification of all required features,

- demonstrate completeness of the correspondence,
- and explicitly identify the status of each requirement.

12.1 Correspondence Audit Grid

Feature	GR Requirement	GETT Mechanism	Status
Metric Structure	Existence of spacetime metric $g_{\mu\nu}$	Emergent effective metric $g_{\mu\nu}^{\text{eff}}$ from Φ-substrate (Section 4, Paper 4.1)	**PASS**
Geometric Framework	Levi–Civita connection, curvature tensors	Constructed from $g_{\mu\nu}^{\text{eff}}$ (Section 7.2)	**PASS**
Einstein Field Equations (Form)	Einstein equation $G_{\mu\nu} = 8\pi G T_{\mu\nu}$	Derived as leading-order closure of Φ–matter dynamics (Section 7.3–7.5)	**PASS**
Stress-Energy Source	Well-defined $T_{\mu\nu}$	Constructed effective tensor from matter + Φ + interaction (Section 5)	**PASS**
Stress–energy Conservation Law	$\nabla^{\mu} T_{\mu\nu} = 0$	Derived conservation of $T_{\mu\nu}^{\text{eff}}$ in Einstein domain + controlled coarse-graining (Section 5.3)	**PASS**
Bianchi Consistency	$\nabla^{\mu} G_{\mu\nu} = 0$	Automatically satisfied by geometric construction (Section 7.2)	**PASS**
Gravitational Constant	Constant (G)	Emergent $G_{\text{eff}} \approx$ const in Einstein domain (Section 7.6)	**PASS**
Weak-Field Limit	Poisson equation $\nabla^2 \Phi = 4\pi G \rho$	Recovered from Einstein-domain equations (Section 9.1)	**PASS**
Newtonian Correspondence	Classical gravity consistency	Matches independent GETT derivation (Paper 2) (Section 9.2)	**PASS**
Vacuum Solutions	$G_{\mu\nu} = 0$	Recovered in Einstein-domain exterior regime (Section 9.3)	**PASS**
Static Symmetric Solutions	Schwarzschild metric behaviour	Schwarzschild-like solution recovered at leading order (Section 9.3)	**PASS**
Locality of Dynamics	Local tensorial field equations	Leading-order local closure; non-local effects in $\Delta_{\mu\nu}$ (Section 7.3–7.4)	**PASS**
Higher-Order Terms	Absent in GR	Explicitly identified in $\Delta_{\mu\nu}$, suppressed in domain (Section 7.4, 8.2)	**PASS**
Domain of Validity	Implicit in GR	Explicit inequalities defining Einstein domain (Section 10)	**PASS**
Failure Conditions	Not specified in GR	Explicit physical breakdown mechanisms (Section 11)	**PASS**
Falsifiability	Implicit/limited	Explicit observational predictions via $\Delta_{\mu\nu}$ (Section 8.4)	**PASS**

Table 11. Correspondence Audit Grid

Table 11 shows the formal correspondence audit between General Relativity and GETT within the Einstein domain, summarising how each required structural, dynamical, and consistency feature of GR is constructed, recovered, and verified as a leading-order consequence of the coarse-grained Φ–matter dynamics. Taken together, the audit confirms that no element of General Relativity is assumed a priori; all required features arise from the underlying Φ-substrate framework under explicitly defined closure conditions.

12.2 Interpretation of Audit

The audit demonstrates that:

- all structural requirements of General Relativity are recovered,
- all dynamical requirements are satisfied at leading order,
- all consistency conditions (e.g. conservation, Bianchi identity) are met,
- and all deviations are explicitly identified and controlled.

Crucially, no required feature of GR is assumed without construction.

12.3 Key Result

The correspondence is therefore complete in the following sense:

General Relativity is recovered as the leading-order dynamical closure of the coarse-grained Φ–matter system within a precisely defined domain, with all departures arising from identifiable and parameter-controlled physical effects.

12.4 Summary Statement

All structural, dynamical, and consistency requirements of General Relativity are satisfied within the Einstein domain of GETT, as verified by the correspondence audit, thereby establishing GR as a fully recovered effective theory within its domain of validity.

13. What This Paper Does NOT Claim

This section explicitly delineates the scope and limitations of the present work, identifying claims that are intentionally not made in this paper.

The objective is to:

- prevent overinterpretation of the results,
- clarify the boundaries of the current derivation,
- and distinguish established results from future work.

13.1 No Claim of Fundamental Replacement of GR

This paper does not claim that General Relativity is incorrect within its domain of applicability.

Rather, it demonstrates that:

- GR is recovered as a leading-order effective theory,
- its predictions remain valid within the Einstein domain,
- and its success is preserved, not negated.

13.2 No Claim of Exact Equality Beyond Leading Order

The correspondence established in this paper is not an exact identity at all scales and conditions.

Specifically, this work does not claim that:

$$G_{\mu\nu} = 8\pi G_{\text{eff}} T_{\mu\nu}^{\text{eff}}$$

holds universally without correction.

Instead:

- corrections $\Delta_{\mu\nu}$ are explicitly present,
- their suppression is conditional,
- and exact equality is achieved only in the ideal Einstein-domain limit.

13.3 No Complete Classification of Exact Solutions

This paper does not provide a full classification or derivation of all solutions to the GETT field equations.

In particular, it does not:

- derive the full family of static or dynamic solutions,
- analyse all vacuum or near-vacuum configurations,
- or exhaustively map the solution space beyond the Einstein domain.

Such work is deferred to future studies.

13.4 No Full Observational Fit Across All Regimes

While this paper identifies observable consequences and testable predictions, it does not claim:

- a complete quantitative fit to all astrophysical or cosmological data,
- a full parameter estimation framework,
- or a comprehensive comparison with all competing models.

The focus here is on establishing the theoretical correspondence, not completing all empirical validation.

13.5 No Exhaustive Treatment of Beyond-Einstein Regimes

Although deviations from GR are identified and classified, this paper does **not** provide a complete dynamical theory for all non-Einstein regimes.

Specifically, it does not:

- fully solve dynamics in low-density or high-gradient regimes,
- develop full cosmological evolution models,
- or analyse ultra-high-density behaviour (e.g. compact object interiors).

These regimes require dedicated treatment.

13.6 No Claim of Universal Constancy of G_{eff}

This work does not assert that the gravitational coupling is universally constant.

Instead:

- G_{eff} is shown to be approximately constant only within the Einstein domain,
- and may vary outside that regime.

Thus, constancy of G is an emergent property, not a fundamental assumption.

13.7 No Claim of Complete Equivalence Principle Universality

This paper does not claim that equivalence-principle behaviour is exact in all regimes.

Rather:

- effective universality emerges in the Einstein domain,
- but deviations may arise when closure conditions fail.

13.8 No Claim of Final or Complete Theory

This work represents a step in the GETT Correspondence Series and does not claim to be a complete or final formulation of gravitational physics.

Instead, it establishes:

- the dynamical correspondence with GR,
- the structure of corrections,
- and the framework for further development.

13.9 Summary Statement

This paper establishes the Einstein-domain dynamical correspondence between GETT and General Relativity without asserting universal equivalence, complete solution classification, or exhaustive empirical validation, thereby clearly distinguishing proven results from areas requiring further investigation.

14. Bridge to Paper 4.3

This section outlines the transition from the theoretical derivation of Einstein-domain dynamics to the empirical validation of those dynamics, defining the objectives and scope of the next stage in the GETT Correspondence Series.

The objective is to:

- connect the formal results of this paper to observable physics,
- define the specific tests required to validate the correspondence,

- and position the next paper (4.3) as the empirical evaluation of the framework established here.

14.1 From Derivation to Test

This paper has established that:

- an effective metric emerges from the coarse-grained Φ–matter system (Paper 4.1),
- the dynamics of that metric admit Einstein-form closure within a defined domain (Section 7),
- all corrections to that closure are explicitly identified and parameter-controlled (Section 8),
- and the domain of validity is precisely specified (Section 10).

The remaining question is therefore:

Does the Einstein-domain formulation of GETT reproduce the full set of observational phenomena successfully described by General Relativity?

14.2 Target Observables

Paper 4.3 will evaluate the Einstein-domain dynamics against key observational tests, including:

14.2.1 Classical Solar System Tests

- perihelion precession,
- light deflection,
- gravitational time delay,
- gravitational redshift.

These tests probe:

- high-density, weak-gradient regimes,
- where $\epsilon \ll 1$ and GR is expected to hold.

14.2.2 Strong-Field and Compact Object Tests

- binary pulsar timing,
- gravitational wave emission,
- compact-object orbital dynamics.

These probe:

- regimes where curvature is strong, but closure conditions may still hold,
- testing the robustness of Einstein-domain behaviour.

14.2.3 Weak-Field and Galactic-Scale Tests

- galaxy rotation curves,
- stellar velocity dispersion profiles,
- gravitational lensing in low-density environments.

These probe:

- transition and non-Einstein regimes,
- where $\Delta_{\mu\nu}$ becomes significant.

14.2.4 Cosmological Behaviour

- large-scale structure formation,
- expansion history,
- density-dependent evolution.

These probe:

- ultra-low-density regimes,
- where coupling modulation and non-adiabatic effects may dominate.

14.3 Core Evaluation Criteria

The correspondence will be assessed against three criteria:

(A) Einstein-Domain Fidelity

- Does GETT reproduce all GR predictions in regimes where $\epsilon \ll 1$?

(B) Controlled Deviations

- Do deviations from GR occur only where predicted by the breakdown conditions?

(C) Predictive Consistency

- Are deviations consistent across independent systems and observables?

14.4 Expected Outcomes

The framework developed in this paper leads to the following expectations:

- **Within the Einstein domain:**
  GETT reproduces GR to observational precision.
- **Near domain boundaries:**
  small, structured deviations appear.
- **Outside the Einstein domain:**
  deviations become significant and potentially explain observed anomalies.

14.5 Strategic Role of Paper 4.3

Paper 4.3 will therefore:

- test the validity of the Einstein-domain closure,
- quantify the magnitude of correction terms,
- and determine whether GETT provides a consistent and predictive description across regimes.

It will serve as the **empirical validation stage** of the correspondence programme.

14.6 Summary Statement

Having established that Einstein's field equations arise as the leading-order dynamical closure of the coarse-grained Φ–matter system within a precisely defined domain, the next step is to evaluate whether this formulation reproduces the full range of observed gravitational phenomena, thereby testing the validity and predictive power of GETT across physical regimes.

15. Conclusion

15.1 Summary of Results

This paper has established the dynamical correspondence between General Expanse Tension Theory (GETT) and General Relativity within a precisely defined physical regime.

Building on the geometrisation limit developed in Paper 4.1, we have shown that:

- the coarse-grained Φ–matter system admits an effective metric description,
- the associated curvature tensors are well-defined and satisfy the required geometric identities,
- a unified effective stress-energy tensor can be constructed and shown to be conserved at leading order,
- and the resulting dynamical equations admit a closure of the form

$$G_{\mu\nu} = 8\pi G_{\text{eff}}\, T_{\mu\nu}^{\text{eff}} + \Delta_{\mu\nu}$$

Under the Einstein-domain conditions, all correction terms are perturbatively suppressed, yielding:

$$G_{\mu\nu} \approx 8\pi G_{\text{eff}}\, T_{\mu\nu}^{\text{eff}}$$

with exact recovery in the formal limit $\epsilon \to 0$.

15.2 Interpretation

The central result of this work is that Einstein's field equations are not fundamental in GETT but arise as the leading-order dynamical closure of the coarse-grained Φ–matter system when the conditions of smoothness, weak coupling variation, adiabatic evolution, and effective universality are satisfied.

In this framework:

- curvature is the geometric encoding of substrate tension dynamics,
- the gravitational coupling is an emergent property of the Φ–matter interaction,
- and the apparent universality of General Relativity reflects the stability of the Einstein-domain conditions.

Thus, General Relativity is recovered not as a primitive assumption, but as a domain-limited effective theory.

15.3 Recovery of Established Physics

It has been demonstrated that, within the Einstein domain:

- the weak-field limit reproduces the Poisson equation of Newtonian gravity,
- the results are consistent with the independent Newtonian correspondence established in Paper 2,
- and the leading-order static, spherically symmetric solutions recover Schwarzschild-like behaviour.

Accordingly, all established gravitational limits are preserved.

15.4 Structure of Deviations

All departures from Einstein-domain behaviour are contained within the correction tensor $\Delta_{\mu\nu}$, which has been:

- explicitly decomposed into physically interpretable contributions,
- shown to scale with well-defined dimensionless parameters,
- and linked to observable deviations in low-density, high-gradient, and non-adiabatic regimes.

This provides a unified and controlled framework for understanding gravitational phenomena beyond the domain of General Relativity.

15.5 Domain of Validity

A precise definition of the Einstein domain has been provided, expressed through explicit inequalities governing:

- coupling variation,
- Φ-field smoothness,
- temporal evolution,
- and universality of matter response.

Within this domain, General Relativity is recovered; outside it, deviations are expected and structured.

15.6 Conceptual Implications

The results of this work suggest a broader interpretation of gravitational physics:

- fundamental dynamics are governed by a real Φ-substrate coupled to matter,
- spacetime geometry emerges as a macroscopic representation of that system,
- and classical gravitational laws arise as effective descriptions valid within specific physical regimes.

This reframes General Relativity as a conditionally valid emergent theory, rather than a universally fundamental one.

15.6 Schematic Overview of the Dynamical Correspondence

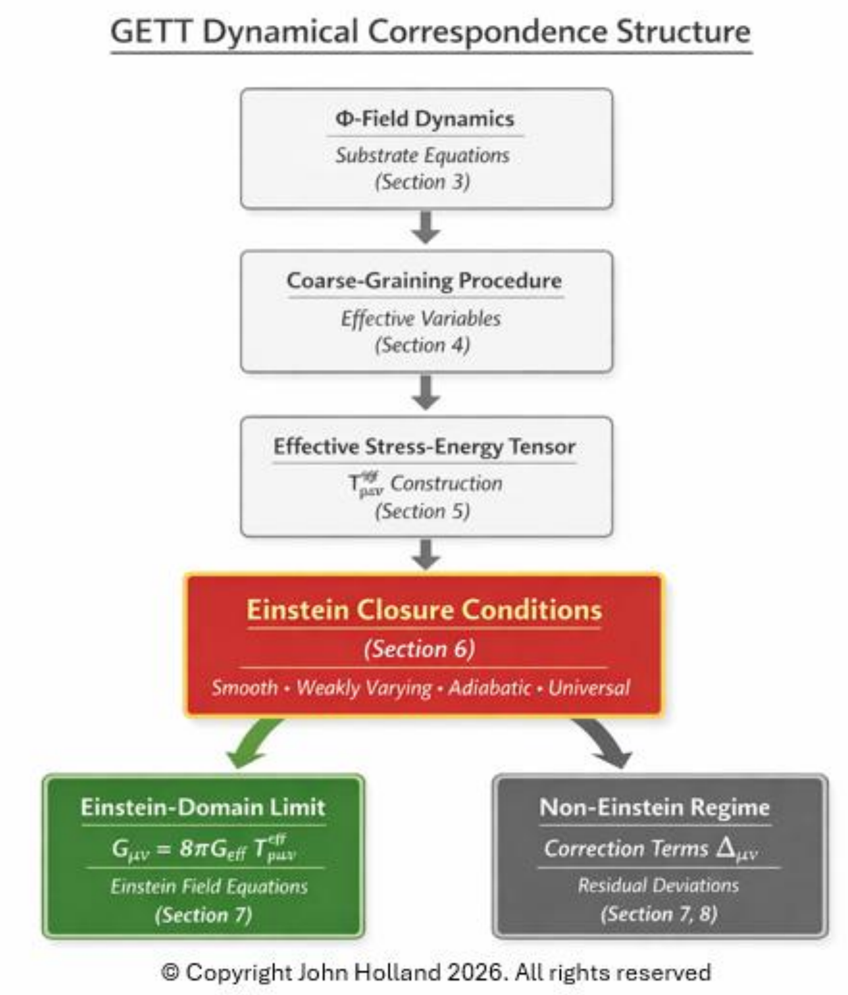

Figure 1. GETT Dynamical Correspondence Structure

Figure 1 shows the structural architecture of the GETT dynamical correspondence established in this work, illustrating the progression from first-principles Φ-substrate dynamics to the emergence of Einstein-domain field equations under explicitly defined closure conditions.

The diagram highlights the layered construction developed throughout the paper. Starting from the governing Φ-field dynamics (Section 3), a controlled coarse-graining procedure (Section 4) yields effective macroscopic variables, from which a unified stress-energy tensor is constructed (Section 5). These steps define the physical and mathematical basis of the emergent continuum description.

At the centre of the structure are the Einstein closure conditions (Section 6), which act as a **gating mechanism** determining whether a local, tensorial curvature–source relation can be established. These conditions – requiring smooth field behaviour, weak coupling variation, adiabatic evolution, and effective universality – define the precise regime in which Einstein-form dynamics emerge.

The diagram makes explicit the bifurcation in behaviour:

- **Within the Einstein domain**, where the closure conditions are satisfied ($\epsilon \ll 1$), the correction tensor is perturbatively suppressed, and the dynamics reduce to the Einstein field equations with an approximately constant effective gravitational coupling.
- **Outside the Einstein domain**, where one or more closure conditions fail, residual contributions encoded in the correction tensor $\Delta_{\mu\nu}$ remain dynamically significant, leading to structured and physically interpretable deviations from General Relativity.

This schematic therefore captures the central result of the paper: that Einstein's field equations arise as the **leading-order dynamical closure of the coarse-grained Φ–matter system**, and that their validity is governed by explicit, physically meaningful conditions. It provides a unified visual summary of both the derivation and the domain-limited nature of the correspondence.

15.7 Outlook

Having established the dynamical correspondence with General Relativity, the next step is to test the Einstein-domain formulation against observational data.

Paper 4.3 will:

- evaluate the reproduction of classical and relativistic gravitational tests,
- quantify deviations predicted by the correction tensor,
- and assess the empirical viability of GETT across physical regimes.

15.8 Final Statement

This work demonstrates that the Einstein field equations arise naturally as the leading-order dynamical closure of the coarse-grained Φ–matter system within a clearly defined domain of validity, thereby providing a physically grounded reconstruction of General Relativity and establishing a framework in which both its success and its limitations are understood as consequences of underlying substrate dynamics.

References

[1] Einstein, A. (1915) *Die Feldgleichungen der Gravitation.*
Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften, 844–847.

[2] Einstein, A. (1916) *Die Grundlage der allgemeinen Relativitätstheorie.*
Annalen der Physik, 49, 769–822.

[3] Hilbert, D. (1915) *Die Grundlagen der Physik.*
Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen.

[4] Landau, L. D. & Lifshitz, E. M. (1975) *The Classical Theory of Fields.*
Pergamon Press.

[5] Wald, R. M. (1984) *General Relativity.*
University of Chicago Press.

[6] Misner, C. W., Thorne, K. S. & Wheeler, J. A. (1973) *Gravitation.*
W. H. Freeman.

[7] Carroll, S. M. (2004) *Spacetime and Geometry: An Introduction to General Relativity.*
Addison-Wesley.

[8] Weinberg, S. (1972) *Gravitation and Cosmology.*
Wiley.

[9] Peskin, M. E. & Schroeder, D. V. (1995) *An Introduction to Quantum Field Theory.*
Westview Press.

[10] Zee, A. (2010) *Quantum Field Theory in a Nutshell.*
Princeton University Press.

[11] Higgs, P. W. (1964) *Broken Symmetries and the Masses of Gauge Bosons.*
Physical Review Letters, 13, 508–509.

[12] Englert, F. & Brout, R. (1964) *Broken Symmetry and the Mass of Gauge Vector Mesons.*
Physical Review Letters, 13, 321–323.

[13] Padmanabhan, T. (2010), Gravitation: Foundations and Frontiers, Cambridge University Press.

[14] Sakharov, A. D. (1967), Sov. Phys. Dokl., 12, 1040.

[15] Jacobson, T. (1995), Phys. Rev. Lett., 75, 1260.

GETT / ETT Development Series

[16] Holland, J. E. (2025). *The Gauge-Invariant Singlet Scalar Field and Its Higgs-Portal Coupling*
https://doi.org/10.5281/zenodo.17794364

[17] Holland, J. E. (2025). *Expanse Tension Theory Hypothesis 3: The Holland–Higgs Field as a Single Scalar Unification of Mass, Gravity, and Cosmic Expansion.* Zenodo.
https://doi.org/10.5281/zenodo.17533253

[18] Holland, J. E. (2025). *Expanse Tension Theory Hypothesis 3 Part B: Density-Dependent Breakdown of Higgs Mass Licensing.* Zenodo. https://doi.org/10.5281/zenodo.17612861

[19] Holland, J. E. (2025). *General Expanse Tension Theory Hypothesis 4: Complete Single-Field Unification via the Universal Coupling Modulation Equation.* Zenodo. https://doi.org/10.5281/zenodo.17665032

[20] Holland, J. E. (2025). *Expanse Tension Theory: A Unified QFT Framework for Gravitation, Inertia, and Cosmic Evolution.* Zenodo. https://doi.org/10.5281/zenodo.16884165

[21] Holland, J. E. (2025). *General Expanse Tension Theory: Universal Field Coupling and the Emergent Nature of Physical Law.* Zenodo. https://doi.org/10.5281/zenodo.16947937

[22] Holland, J. E. (2026). *Classical Kinematics in the Weak-Field Density Regime: A GETT Reconstruction* (GETT Correspondence Series Paper 1). https://doi.org/10.5281/zenodo.18882300

[23] Holland, J. E. (2026). *Newtonian Gravity in the Density-Suppressed Weak-Field Regime: A GETT Reconstruction.* (GETT Correspondence Series Paper 2) https://doi.org/10.5281/zenodo.19000660

[24] Holland, J. E. (2025). *Theoretical Potential for Expanse Tension Ripples as the Energy Reservoir Driving Cosmic Expansion.* Zenodo. https://doi.org/10.5281/zenodo.17152332

[25] Holland, J. E. (2025). *The Orthogonal Emergence of Time, Inertia, and Gravity within General Expanse Tension Theory (GETT)* https://doi.org/10.5281/zenodo.17880426

[26] Holland, J. E. (2026). *The GETT Covariant Scalar-Density Framework for Emergent Gravity and Cosmology.* https://doi.org/10.5281/zenodo.19037600

[27] Holland, J. E. (2026). *Density-Regulated Scalar–Matter Coupling via the Higgs Portal: Distinction, Correspondence, and Unification Across Scalar Field Theories.* https://doi.org/10.5281/zenodo.19181843

[28] Holland, J. E. (2026). *GETT Correspondence Paper 3: Reconstruction of Special Relativity as an Emergent Local Kinematic Limit*. https://doi.org/10.5281/zenodo.19235547

[29] Holland, J. E. (2026). *GETT Correspondence Series Paper 4.1 – General Relativity Correspondence I: Emergent Metric Structure and the Geometrisation Limit.* https://doi.org/10.5281/zenodo.19280759